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EVALUATION OF PARAMETERS DYNAMIC OBJECT WHEN AVAILABLE LIMITED INTERFERENCE

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Modeling tasks play an essential role in the analysis and synthesis of economic end technical processes. These processes include, first of all, tasks of planning, control and management. Most economic systems operate in conditions when the requirements for the management processes are set in the form of a system of so-called target inequalities. In this case, if a system of inequalities is specified with respect to the object and the control goal, the violation of which is unacceptable, then the corresponding control law is called critical, and the control system that implements it is called critical.

Keywords: *modeling, process, identification, control, ellipsoidal estimation, criteria, algorithm.*

INTRODUCTION

One of the fundamental problems of control theory is the problem of identification - determination or estimation of system parameters at different points in time. The quality of the solution to the identification problem significantly depends on the amount of a priori information about the properties of the object under study and the acting signals and noises. Most of the existing identification methods assume the presence of such information in the form of a known noise distribution density or information about the belonging of an unknown density to some class of distributions. This information makes it possible to unambiguously select an identification criterion and apply well-developed methods to find its extremum.

However, there is often no information about the statistical properties of signals and interference, and the researcher has information only about their levels. This work is devoted to the study of such a case.

PROBLEM ANALYSIS

In problems of computer engineering and control, a situation often arises when a closed-loop control system, which is under the influence of an external disturbing signal (external, reference signal, interference, signals from other objects, variations in environmental parameters, etc.), must maintain the characteristics of the object. control (object output signal, control error, etc.) within some a priori set boundaries so that

$$|v(t, w)| \le E \quad \forall t \in R,$$

where t is continuous or discrete time. In the event that violation of inequality is in principle unacceptable, for example, leads to catastrophic consequences, the control law that ensures the strict maintenance of this inequality is called critical, and the control system that implements it is called critical [1, 2].

In everyday practice, critical management tasks are encountered quite often, and among the most typical are the following:

- in the tasks of air traffic control, the aircraft must constantly be inside a rather narrow air corridor, leaving the boundaries of which, in principle, is not permissible [3];
- a catalytic converter, the presence of which in a car is required by the legislation of most civilized states, works effectively only in situations where the characteristics of the air-fuel mixture are maintained within tight boundaries;
- in telecommunication systems, the tracking accuracy by the communication satellite system is set in the form of a narrow error range [4];
- in biomedical systems, the control parameters of the controlled organism must be within the boundaries that guarantee stable vital activity.

Since any real system is subject to the influence of many controlled and uncontrolled disturbances and disturbances, the purpose of the critical system is to maintain the output signals of the object within the specified boundaries, regardless of the nature of these disturbances.

In the general case, the problem of maintaining the output signals of the control object within the given boundaries arose quite a long time ago, and a number of approaches were developed to solve it. So, in [5] a statistical approach was proposed that maximizes the probability that the outputs of the object will not go beyond certain boundaries for arbitrary random inputs. There is a known method based on the set-theoretic approach [6-8], using the concept of a “target tube”, inside which the phase variables of an object under the influence of unknown but restrictive disturbances must remain. An efficient computational algorithm implementing this approach was proposed in [9], and in [10] the solution to the problem was extended to nonlinear multidimensional objects.

In the general case, the goal of any feedback control system is to ensure the required behavior of an object by appropriate processing of input and output signals, calculation of control actions and their delivery to the executive bodies. The main problem in this case is the projection of the regulator itself, from a theoretical point of view, it is a formal algorithm, the result of which is the numerical value of the control signal.

The synthesis problem splits into two relatively independent subtasks, the first of which is to determine the goal of control and its formal presentation - a criterion. The second subtask is to find a formal description of the regulator that provides the required value of this criterion. In addition to the main criterion, a number of additional subgoals are usually introduced into consideration, requiring, for example, that the closed-loop system be stable, the control signals or some functions from them would not be too large, the effects of noise and disturbances would be small, and the system itself would tend to the required state of some in a certain way.

At the same time, when designing, many criteria and subgoals are usually considered, many of which are competing or even contradictory. Therefore, when designing, it is extremely important to be able to take into account the trade-off between different criteria. It is important to note that although to date, within the framework of control theory, many different criteria have been proposed, there is no universal criterion that takes into account all possible requirements for the quality of processes occurring in the system. Therefore, the developer of the control system must choose a criterion or criteria that take into account his often subjective ideas about how this system should behave. Moreover, for an arbitrary criterion chosen at random, there is always a control algorithm that provides an extremum for this criterion. In practice, however, usually used criteria related to the accuracy of regulation or tracking, efficiency in terms of speed, performance, noise immunity, costs, stability, etc. These criteria usually represent some functions of the input and output signals or states of the control object, while the signals are usually assumed to be stochastic processes with some a priori given probabilistic structure. The most commonly used hypothesis is the Gaussian distribution of useful signals and interference. It is on this hypothesis that the popular LCG problem [11] is based, H^2 - the optimization problem [12] and the classical stochastic control theory [13-15]. In practical problems, Gaussian processes are not so common, and their use is mainly associated with mathematical convenience.

The H^∞ optimization problem [16, 17], which requires only that the signals of the object be square integrable and have limited energy, is associated with less restrictions on the statistical nature of the signals.

Great flexibility in the design of control systems can be achieved by assuming that the signals belong to a certain functional space. Such a space can be determined by setting boundaries for the amplitudes, rate of change, energy, and other characteristics of the signals. Such a description of signals is much simpler than a statistical one, has a clear physical meaning and, in general, facilitates the process of designing a control system.

The choice of the control criterion, or the objective function, is the starting point for the synthesis of the control algorithm itself, which provides the required value for this criterion. To date, control theory has developed three main approaches to the synthesis of control systems:

- algorithmic methods,
- optimization methods,
- method of inequalities.

A number of methods are synthetic in the sense that they use combinations of these approaches. So, for example, algorithmic methods can use optimization procedures at separate stages of synthesis. Such algorithms were proposed, for example, in [18-20]. Some of the approaches, such as the zeros and poles placement method, do not apply to any of the approaches mentioned.

Algorithmic methods are characterized by the absence of a single integral criterion. They are based on the execution of a sequence of stages, at each of which different, sometimes contradictory, subgoals are realized. In this case, the system is divided into subsystems, each of which is optimized according to its own local criterion.

Optimization methods are much more rigorous from a mathematical point of view; as a result of their application, a certain general optimization criterion J is minimized. Within this approach, two areas can be distinguished: analytical methods and parametric optimization.

The main advantage of analytical methods is that the optimality of the obtained solution in the sense of the adopted criterion is guaranteed. The disadvantage is that the resulting algorithms are often complex, have a high order and, in the general case, can lead to instability of the system as a whole. This disadvantage is usually overcome by using less complex regulators, while providing insignificant loss of control quality.

The inequality method is the most prominent representative of the multipurpose approach to the synthesis of control systems. It operates simultaneously with a variety of local criteria, providing each of them with a value no worse than a certain threshold value. In the original inequality method [21], all requirements for the quality of control processes are specified in the form of a system of inequalities of the form

$$J_i(p) \le \varepsilon_i \ \forall i = 1, 2, ..., n ,$$

where local criteria $J_i : p \rightarrow J_1(p)$ perform mapping; $R^N \rightarrow R$; $p \in P$, $p = (p_1, p_2, ..., p_N)$ – N - dimensional vector of controlled variables; ε_i - selectable threshold values of individual objective functions.

In this case, the control goal is considered achieved if all the n inequalities of the given system are satisfied.

In our opinion, today the inequality method is the most promising approach that allows one to take into account a variety of very diverse, often contradictory requirements for the control system, and minimizes the element of subjectivity associated with the construction of a single local optimization criterion. At the same time, this method requires its further improvement and development, determined by the need to work under conditions of uncertainty about the object and the environment (adaptive properties), non-stationarity, nonlinearity, the need to develop control actions in real time.

FORMULATION OF THE PROBLEM

Consider a dynamic object described by an autoregressive (ARX) model

$$y_t = a_1 y_{t-1} + \ldots + a_n y_{t-n} + b_0 u_1 + b_1 u_{t-1} + \ldots + b_m u_{t-m} + \xi_t, \tag{1}$$

where y_i, u_i, ξ_i are the output, input signals and noise at the moment of time i, respectively,

$$|\xi_i| \le \delta_i \ , \quad i = 1,2,\ldots,t . \tag{2}$$

This equation can be transformed to the form

$$y_t = \Theta^T x_t + \xi_t, \tag{3}$$

where $\Theta = (a_1, a_2, \ldots, a_n, b_0, b_1, \ldots, b_m)^T$ is the vector of parameters; $x_t = (y_{t-1}, \ldots, y_{t-n}, u_t, u_{t-1}, \ldots, u_{t-m})^T$ is a vector of generalized inputs.

Taking into account (2), equation (3) can be rewritten as a pair of inequalities

$$y_t - \delta_t \le \Theta^T x_t \le y_t + \delta_t, \tag{4}$$

defining the boundaries of the domain D, inside which the required parameters Θ must lie. Note that inequalities (4) define hyperplanes in space Θ that bound the domain of membership. The sequence of observations $y_1, y_2, \ldots, y_k$ generates k pairs of hyperplanes, "cutting out" in space a certain area D_k, which is the area of estimates and is a polytope [1].

Each new observation changes this area, relative to which it can be noted that all points belonging to this area are equal in the sense that the best estimate cannot be distinguished among them. Therefore, for convenience, a certain "center" of the region is used [1, 2].

Obviously, the larger the volume of the obtained polytope, the lower the level of uncertainty Θ. It can be seen from (3) and (4) that the type and size of the polytope depends on the choice of the vector of generalized inputs x_t. When used as a metric in the parameter Θ space of the Euclidean distance, the best choice x_t is associated with maximization $\|x_t\|$, which ensures the maximum contraction of the boundaries of the polytope [3]. Thus, the problem of choosing a sequence of vectors x_t consists in minimizing the size of the region D_k after k steps.

OBTAINING AN ESTIMATION ALGORITHM

Formally, this can be represented as follows. Since the object is described by equation (3), and the noise satisfies condition (2), the vector of the sought parameters satisfies all inequalities

$$\left(y_t - \Theta^T x_t\right)^2 \le \delta_t^2 . \tag{5}$$

Therefore, as parameter estimates, only those that belong to the set

$$M_t\left(\hat{\Theta}\right) = \left\{\hat{\Theta} : \left(y_t - \hat{\Theta}^T x_t\right)^2 \le \delta_t^2, \hat{\Theta} \in R^{n+m+1}\right\}, \tag{6}$$

which, from a geometric point of view, is a monotone nonincreasing sequence of convex polytopes D_t:

$$M_t = \bigcap_{k=1}^{t} D_k = M_{t-1} \cap D_t ; \tag{7}$$

$$D_t = \left\{\hat{\Theta} : \left|y_t - \hat{\Theta}^T x_t\right| \le \delta_t\right\}. \tag{8}$$

The computation of estimates $\hat{\Theta}_t \in M_t(\hat{\Theta})$ is a complex problem, the solution of which can be greatly simplified by constructing a certain bounding set $M_t(\hat{\Theta})$. These constraints can be specified either in the form of parallelepipeds or in the form of ellipsoids, the center of which coincides with Θ.

Consider an object identification algorithm (1) based on the ellipsoid method.

The algorithm begins by constructing a sufficiently large ellipsoid M_0 in space R^{n+m+1} and containing all possible admissible values of the vector Θ. After obtaining the first observation y_1, an ellipsoid can be found constructed in accordance with (7) at the intersection M_0 of the convex polytope D_1. To accelerate the convergence of the algorithm, the ellipsoid should be optimized, for example, according to the criterion of its minimum volume or minimum trace.

Let's denote the optimal ellipsoid as M_1. After obtaining the second observation y_2, we will find M_2, etc. in a similar way. Thus, a sequence of optimal ellipsoids can be obtained. At an arbitrary time, moment t, the ellipsoid is determined by expression (6) or, more generally, by the expression

$$M_t = \left\{\hat{\Theta} : \left(\Theta - \hat{\Theta}_t\right)^T P_t^{-1}\left(\Theta - \hat{\Theta}_t\right) \le r_t^2\right\}, \tag{9}$$

where P_t is the weight matrix defining the semiaxes of the ellipsoid; r_t^2 is a scalar value, recursively calculated by the formula

$$r_t^2 = (1+\lambda_t)r_{t-1}^2 + \lambda_t\delta_t^2 - \frac{\lambda_t(1-\lambda_t)e_t^2}{1-(1-\gamma_t)\lambda_t}, \tag{10}$$

$$r_0^2 >> 1;$$

$$e_t = y_t - \hat{\Theta}_{t-1}^T x_t;$$

$$\gamma_t = x_t^T P_{t-1} x_t,$$

$$\lambda_t \in (0,1].$$

The estimation is corrected according to the formula

$$\hat{\Theta}_t = \hat{\Theta}_{t-1} + \lambda_t \frac{P_{t-1}x_t}{1-(1-\gamma_t)\lambda_t} r_t; \tag{11}$$

$$P_t = \frac{1}{1-\lambda_t}\left[P_{t-1} - \lambda_t \frac{P_{t-1}x_t x_t^T P_{t-1}}{1-(1-\gamma_t)\lambda_t}\right], \tag{12}$$

where $P_0 = \alpha I$, I -identity matrix, $\alpha >> 0$.

The size of the optimal ellipsoid M_t calculated in accordance with (7) depends on the coefficient λ_t. The optimal value λ_t can be obtained by minimizing λ_t the value r_t^2 (10).

On the other hand, it makes sense to carry out the correction λ_t only at those moments of time when the estimates of the sought parameters are refined, i.e., in cases when condition (5) is violated.

Taking into account (10), this condition can be modified as follows:

$$\lambda_t = \min\{\alpha, \beta_t\}, \tag{14}$$

$$\beta_t = \begin{cases} \alpha \quad \textit{at} \quad \Delta_t = 0 \\ \text{or} \quad (1+\Delta_t(\gamma_t - 1)) \le 0; \\ \dfrac{1-\Delta_t}{2} \quad \textit{at} \quad \gamma_t = 1; \\ \dfrac{1}{1-\gamma_t}\left[1 - \left(\dfrac{\gamma_t}{1+\Delta_t(\gamma_t-1)}\right)^{\frac{1}{2}}\right] \quad \textit{at} \quad 1+\Delta_t(\gamma_t - 1) > 0; \end{cases} \tag{15}$$

$$\Delta_t = \frac{\delta_t^2 - r_{t-1}^2}{e_t^2}.$$

CONCLUSION

Thus, an identification algorithm is obtained, which is described by relations (11), (12), (14), (15). The value chosen as the upper bound δ_t should not be closely related to the magnitude of the interference, since the estimates of the interference boundaries do not affect the estimates of the sought parameters. However, overestimation δ_t can lead to an increase in the size of the bounding ellipsoid, and underestimation can lead to a decrease (and even the appearance of negative values) δ_t^2, which leads to a decrease or disappearance of the bounding ellipsoid. In the latter case, you should either artificially increase the size of the ellipsoid M_t, or increase the width by increasing the value δ_t.

The considered algorithm is some modification of the recurrent least squares method with exponential weighting of information. The computational procedure makes it possible to solve the identification problem without any special difficulties.

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