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A BAYESIAN MODEL FOR MEASUREMENT BY COUNTING AND ITS APPLICATION TO CONFORMITY ASSESSMENT

Pennecchi F., Bich W.,
Istituto Nazionale di Ricerca Metrologica (INRiM), Torino, Italy

Counting objects or events (hereafter simply "items") occurs very often in many scientific and technological domains, being at the base of many high-level measurements in fields such as, for example, time and frequency, optics, ionizing radiations, microbiology and chemistry. Also, in everyday life, countings play a fundamental role, as for the use of electric power meters, based on counting a number of impulsions, the assessment of pre-packaged goods, whose total price depends on the number of exchanged items, the monitoring of medication consumption, for example by monitoring of pill intake into blisters.

As for any measurement, the result of a counting process should be expressed as: a) an estimate (the number of counted items) of the measurand (the true number of items) and an associated uncertainty (or a suitable coverage interval), according to JCGM 100:2008 (the legacy GUM [1]); or b) a probability distribution (discrete, in the case of counting) describing the state of knowledge on the measurand, according to JCGM 101:2008 [2]. However, neither of the above-mentioned documents provides adequate guidance for measurements by counting. Furthermore, and despite a large number of publications available on the counting topic in so many different fields and applications, we could hardly find studies aimed at correcting counting errors possibly affecting the measurand estimate and at evaluating the associated uncertainty.

In the present work, we tackle the problem by resorting to a Bayesian model, which allows determining a posterior probability mass function for the measurand while taking into account the probability of over- and under-counting errors. The model is able to deal with any number of false positives and negatives, correcting the estimate of the measurand for the bias due to such errors and associating a proper measurement uncertainty to it. To that aim, we propose a specific likelihood function that accounts for over- and under-counting errors, and does so in terms of two parameters: the (conditional) probability that a non-existing item is mistakenly counted (probability of false positive $p_{\leftarrow}$), and the (conditional) probability that an existing item is not counted (probability of false negative $1 - p_{\rightarrow}$).

Aside from directly applying this model to counting processes, the proposed framework could well serve to classification problems in general, where $p_{\leftarrow}$ and $1 - p_{\rightarrow}$ are related to probabilities of false positives and negatives, respectively. Specifically, it can be applied to the conformity assessment of a sample of items, seen as binary classification problem. JCGM 106:2012 [3] provides a Bayesian framework for the conformity assessment of the true value of a property of interest of a single item, by combining prior knowledge on the measurand with the knowledge acquired in the measurement (affected by measurement uncertainty). Global consumer risk R_c (false positive error) and producer risk R_p (false negative error), defined therein for a single item at a time, can be generalized to the corresponding risks for the whole sample of items. To assess the quality of the sample, indeed, the relevant question to be addressed should be: "given that a certain number of items in the sample are accepted as conforming (since they have showed a measured value of the property of interest within its acceptance interval), which is the probability to actually have the same or another number of truly conforming items (i.e., having their true value within its tolerance interval) in the whole sample"? The proposed modelling is able to answer the question, by appropriately expressing probability parameters $p_{\leftarrow}$ and $1 - p_{\rightarrow}$ in terms of R_c and R_p. This could help in the fields of quality control and inspection, acceptance sampling and conformity assessment.

In conclusion, the proposed model for measurements by counting accommodates any number of under- and over-counting's, naturally considering any prior knowledge on the measurand. It can be generalized to hierarchical models including possible prior knowledge on the statistical parameters involved and can be applied to classification and conformity assessment problems.

References

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