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OBTAINING VALUES AND UNCERTAINTIES OF EQUIVALENT CIRCUIT PARAMETERS OF RESISTORS AND CAPACITORS FROM MEASUREMENTS OF THEIR IMPEDANCE FREQUENCY CHARACTERISTICS

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The paper will present the determination of the parameters *R, L, C* of the equivalent scheme of impedances of resistors and capacitors and also their uncertainties.

So, in general the impedances $\boldsymbol{Z}$ in Cartesian system of such scheme are complex:

$$\boldsymbol{Z} = \mathrm{Re}(\boldsymbol{Z}) + j\,\mathrm{Im}(\boldsymbol{Z}) \tag{1}$$

where $\mathrm{Re}(\boldsymbol{Z})$ is real and $\mathrm{Im}(\boldsymbol{Z})$ is imaginary part- reactance of impedance and j is imaginary unit.

The one of the most important metrological characteristics is the frequency characteristic of transmittance (Body or Nyquist) of such circuits. The examples of frequency characteristic of impedance measured with uncertainties in n points $\mathrm{Re}[\boldsymbol{Z}(\boldsymbol{p},\omega_i)]$, $\mathrm{Im}[\boldsymbol{Z}(\boldsymbol{p},\omega_i)]$ or $|\boldsymbol{Z}|(\boldsymbol{p},\omega_i)$, $i = 1,2,...,n$ are real or imaginary parts or module of impedance depends on all parameters $\boldsymbol{p} = [p_1, p_2, ..., p_m]$ of characterizing elements of circuits and frequency $f = \omega/2\pi$ of the sinusoidal forcing signal are given.

In previous papers, the adjustment of system parameters using Monte Carlo without evaluation of uncertainty of parameters was demonstrated. The simpler method based on WTLS method for fitting straight line with exchanging coordinates is applying to all parameters with uncertainties and determinates the coverage corridor of module of impedance in such frequency characteristic.

In the case of the latter, the uncertainties of these parameters will also be estimated using the least squares method for matching the frequency characteristics of the impedance of the tested element. This task is mathematically complicated because models of the considered passive elements lead to nonlinear equations that do not have analytical solutions in general. The method of fitting with the change of variables leading to linear characteristics will be used. It facilitates the solution of the problem. The proposed method has already been presented on Inter-university Conference of Metrologists MKM2025 in Poznan Pl and on MathMet in Göteborg as poster and has been appreciated by metrologists on these both their meetings.

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