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MEASUREMENT OF STATIC AND DYNAMIC YOUNG'S MODULUS OF THIN FIBERS

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PART I. Measurement of Static Young's Modulus of Thin Fibers

The study of the static Young's modulus of thin fibers by diffraction will be described [1]. The vertical movement in the measuring device is made possible by the air bearing of its own design. Fiber is loaded with a mass *m* and an elongation occurs, which causes a change in the width of the gap. The distribution of the diffraction bands is recorded by a CCD camera.

The function of the distribution of light intensity after passing through a gap is described by the formula:

$$I = I_0 \left[\frac{\sin(k \sin \alpha)}{k \sin \alpha} \right]^2 + I_c . \tag{1}$$

The value of Young's modulus tested for fibers is calculated on the basis of the formula:

$$E = \left(\frac{F}{S} \right) \Big/ \left(\frac{\Delta l}{l} \right), \tag{2}$$

where F – the loading force of the fiber, S – the cross-sectional area of the fiber, Δl – the elongation of the fiber, equal to the change in the width of the gap (ΔD), l – the length of the fiber.

The width of the gap is

$$D = D_0 + \Delta D , \tag{3}$$

where: D_0 – Pre-gap width.

After transforming the formula (2) valid for a fiber with a diameter of $2r$, the following is obtained:

$$E = \frac{F\,l}{\Delta D\,S} = \frac{mgl}{\Delta D\,\pi r^2} = \frac{mgl}{\pi r^2 \left(\frac{k\lambda}{\pi} - D_0 \right)} , \tag{4}$$

hence:

$$k = \frac{gl}{E\lambda r^2} m + \frac{\pi D_0}{\lambda} . \tag{5}$$

Using the results of the fit straight-line $k = am + b$ (where a – the slope parameter of this line), we get the expression for Young's modulus E:

$$E = \frac{gl}{a\lambda r^2} , \tag{6}$$

in which g – acceleration of the earth.

Example 1. Measurement result for the basalt fiber with a length of 93 mm and a diameter of 17,1 µm. The value of Young's modulus E=(27,3±2,1) GPa was determined with an uncertainty of 8,6%.

PART II. Measurements of the Dynamic Young's Modulus of Thin Fibers

The implementation of the measurement of the dynamic modulus of elasticity using laser techniques is presented. A pulsed mechanical spectrometer IMS was developed and built for this purpose. It is measured by Young's modulus under dynamic conditions. Its structure is described in the full version of the thesis. The diffraction optical method used here is highly sensitive, inertial, and non-contact.

The initial values of the parameters were analyzed using a graphical method, plotting several dozen periods of mechanical loss spectra and the matching function on one graph. You can then change the parameters (A, β, n, ω, φ) overlapping two graphs. After entering the obtained initial values of the parameter into the program, an exact match was obtained. The quality of this fit was graphically analyzed by comparing the mechanical loss spectra on a single graph. The method of least squares was also used. If the matched value of the parameter $n = 1$ and the amplitude decay is exponential, then the matching function is described by the formula

$$g(t) = Ae^{-\beta t}\sin(\omega t + \varphi).$$

The obtained parameters β, ω allow the calculation of Young's modulus E and dimensionless logarithmic damping decrement. The calculation omits the mass of the fibers, as it is negligible in relation to the weight of the load.

The formulas describing the above measurements contain the following parameters:

$$\sigma = \frac{F}{S},\ \varepsilon = \frac{\Delta L}{L},\ k = \frac{F}{\Delta L},\ \omega^2 = \frac{k}{m},\ \omega^2 = \omega_0^2 - \beta^2,$$

where: σ – stress, Pa; F – force, N; S – fiber cross-sectional area, $S=\pi D^2/4$, m^2; m – load, kg; ε – relative elongation; L – fiber length, m; k – elasticity coefficient, 1/m.

The dynamic Young's modulus and the logarithmic damping decrement δ were calculated from the following dependencies:

$$E_{IMS} = \frac{\sigma}{\varepsilon} = \frac{FL}{S\Delta L} = k\frac{L}{S} = m\frac{4L}{\pi D^2}(\omega^2 + \beta^2).$$

Example 2. Measurements were made for a polyamide fiber with a length of 30 mm and a diameter of 119 µm. The value of Young's modulus E=(5,84±0,28) GPa was determined with an uncertainty of 4,8%.

The full version of the work also includes:

- introduction to diffraction methods;
- summary and conclusions of the content;
- a list of 20 items of literature in English and Polish cited in the content.