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MATHEMATICAL MODELS OF THE CYCLIC WORK PACKAGE DISTRIBUTION TASK

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The solution to the task of improving the efficiency of technologies for the distribution of work packages on the set of indicators, taking into account the workload of the executors has been obtained. The technology of distribution and execution of work as a process of functioning of three-phase multi-channel mass service system is proposed. The discipline of applications service in such a system is determined by solving the task of work distribution between the channels, and the results of modeling of its functioning cycles allow us to determine the channel occupancy by the time of distribution of the current work package. Mathematical models of single-criteria and multi-criteria tasks with target functions of financial (material), time costs and quality of work are proposed. Double-criteria tasks with different combinations of local criteria are partial variants of the multi-criteria task model. The parameters of the proposed models allow taking into account channel occupancy at the moment of distribution of the current work package and costs for their readjustment after previous work performance. In multicriteria models, it is proposed to use an accuracy-complexity effective function of utility of local criteria and universal additive-multiplicative convolution of criteria on the basis of Kolmogorov–Gabor function. The practical use of the proposed models will allow in practice to obtain the more effective solutions to the tasks of their distribution by taking into account the employment of channels and the cost of their reconfiguration after the previous work.

Introduction

The growing complexity of objects used in all areas of human activity, respectively, complicates the processes of automation of project management of their creation, modernization, re-engineering and technology management. In modern automated technologies, the methodology of the system approach is widely used, which provides for decomposition of the relevant processes into complexes of works and then individual works. The work thus separated is distributed among the executors, which are separate devices, specialists, divisions or companies.

The most large-scale example of work decomposition and distribution can be global production networks, in which the organization of production activities of multinational manufacturing or IT corporations takes place [1]. For their implementation, a methodology of work decomposition based on the results or phases of project management is used, based on the results of which project teams are formed [2–3]. Subsequently, the decomposition of separate project management tasks with the definition of executors or methods of their solution is carried out [4–6]. Depending on the peculiarities of the objects and tasks of the study, such work can be executed in parallel or with connections between them. Regardless of this, there is a need to distribute effectively the work between potential executors under the established constraints on the indicators of quality of results and resource costs [7]. The tasks of work allocation in many cases can be reduced to the classical assignment task or assignment tasks with additional requirements (scheduling of flights, construction work, production and repair work, development of software systems, etc.) [8].

In the classical assignment task, its solution is found for an equal number of works and executors only on the basis of the cost indicator:

$$\begin{cases} f(x) = \sum_{i=1}^{n} \sum_{j=1}^{n} a_{ij} x_{ij} \to \min, \\ a_{ij} > 0,\ i, j = \overline{1,n}, \\ \sum_{i=1}^{n} x_{ij} = 1,\ j = \overline{1,n};\ \ \sum_{j=1}^{n} x_{ij} = 1,\ i = \overline{1,n};\ x_{ij} \in \{0,1\},\ i, j = \overline{1,n}, \end{cases} \tag{1}$$

where n is the number of works and the number of executors; a_{ij} – the time, material or financial cost of doing the i-th work by the j-th executor.

At the same time, the peculiarities of many practical tasks do not satisfy the requirements of model (1):

– because the number of works may not be equal to the number of executors, the matrices $a = \left[a_{ij} \right]$ and $x = \left[x_{ij} \right]$ are not square $\left(i = \overline{1,n},\ j = \overline{1,m},\ n \neq m \right)$;

– there may be prohibitions on the assignment of works;

– the goal may be to find the maximum of a target function, such as profit or quality of work $f\left(x\right) \to \max$;

– the matrix $a = \left[a_{ij} \right],\ i, j = \overline{1,n}$ may contain negative elements;

– several target functions are possible (multi-criteria assignment task).

The task in which the first four features are taken into account is called the general view assignment task. By executing preliminary transformations of the matrix $a = \left[a_{ij} \right]$, the Hungarian method can be used to solve it.

Taking into account the factors of narrow specialization of executors, different levels of their qualification and degree of workload, which are found in many practical situations, leads to the necessity of joint solution of the tasks of determining the workload of executors and distribution [9–10]. In particular, in [11–12] the dynamic task of assigning only one job with time constraints on the maximum efficiency index is considered, for which a model based on the interaction of intelligent agents is proposed. Considering this, the scientific and applied task of increasing the efficiency of work distribution technologies by developing mathematical models of cyclic distribution of work packages on a set of indicators, taking into account the workload of executors, is relevant.

Statement of the task

The object of the study are organizational, technical and organizational-technical systems designed to execute work packages. The subject of the study is the processes of distribution and execution of work packages, taking into account the resource load of the system.

This paper considers a generalized version of the task of modeling the process of executing work packages, which may arrive at random moments of time and require each of the work to execute a random amount of resources (qualification of the executor, time of execution, type of equipment, etc.). Taking into account modern approaches to modeling, such objects of research can be considered as three-phase multi-channel systems of mass service (systems with queues). The system receives applications at random points in time, any of which corresponds to a work package. The first phase channel executes distribution of works, the second phase channels execute works according to their specialization, and the third phase channel combines the results of works of the second phase channels.

A variant of the dynamic task of assigning channels to a mass service system in this formulation is considered.

Defined:

– system structure (first-phase channel, n second-phase channels functioning in parallel, each doing a separate work $j=\overline{1,n}$, and third-phase channel);

– the incoming flow of n work requests (packages), characterized by the law of distribution of intervals between them and its parameters;

– material or financial costs of executing the works of each specialization by each of the channels $c=\left[c_{ij}\right], i,j=\overline{1,n}$;

– distribution laws and their parameters for the duration of work for each specialization by each of the channels τ_{ij}, $i,j=\overline{1,n}$;

– the quality of work of each specialization by each of the channels q_{ij}, $i,j=\overline{1,n}$.

It is necessary: for a given interval of the system in the dynamics to execute cyclically the best distribution of n work among the n channels of the second phase on the indicators of financial costs $k_1 \to \min$, time spent on the execution of work $k_2 \to \min$ and quality of performance of the entire package of work $k_3 \to \max$.

To solve the task it is necessary to develop mathematical models of single-criteria and multi-criteria task of work distribution and their parametric synthesis, involving the determination of material, financial or time costs associated with the cyclic performance of work.

Mathematical models of single-criteria work distribution tasks

Let's consider the task of assigning n channels of the system to execute n work on the indicators of material or financial costs $k_1(x) \to \min$, the cost of time for work (efficiency) $k_2(x) \to \min$ and quality of performance of the entire package of work $k_3(x) \to \max$.

A mathematical model of the assignment task in terms of material and financial costs, taking into account the cost of transition to the current work c_{ij}^{o}, $i, j = \overline{1,n}$ can be presented in the following form:

$$\begin{cases} k_1(x) = \sum_{i=1}^{n} \sum_{j=1}^{n} \left(c_{ij}^{o} + c'_{ij} \right) x_{ij} \to \min, \\ \sum_{i=1}^{n} x_{ij} = 1,\ j = \overline{1,n};\ \ \sum_{j=1}^{n} x_{ij} = 1,\ i = \overline{1,n};\ x_{ij} \in \{0,1\},\ i, j = \overline{1,n}, \end{cases} \tag{2}$$

where c'_{ij} – is the nominal material or financial cost of doing the i-th work by the j-th channel of the system; $x = \left[x_{ij} \right]$, $i, j = \overline{1,n}$ – assignment matrix (Boolean variable $x_{ij} = 1$ if the i-th job is assigned to a i-th executor; $x_{ij} = 0$ – otherwise).

In contrast to the traditional task, in order to set the cost value $c_{ij} = \left(c_{ij}^{o} + c'_{ij} \right)$, $i, j = \overline{1,n}$, it is necessary to take into account what work was executed by the j-th channel in the previous cycle.

Depending on the peculiarities of the task, the values of total time of work execution, time of maximum work duration, duration of critical path activities of the project, etc. can be used as indicators of system $k_2(x)$ operability.

The target function of system operability for technology of sequential execution of a package of independent works has the form:

$$k_2(x) = \sum_{i=1}^{n} \sum_{j=1}^{n} \left(\tau_{ij}^{o} + \tau'_{ij} \right) x_{ij} \to \min, \tag{3}$$

where τ_{ij}^{o} – delay of the j-th channel to complete the preliminary work; τ'_{ij} – time of the i-th job execution by the j-th channel.

As an indicator of system efficiency for the technology of parallel execution of a package of independent works, it is proposed to choose the time of execution of the maximum duration of the work. In this case, the target function of system operability can be represented in the following form:

$$k_2(x) = \max_i \left\{ \left(\tau_{ij}^{o} + \tau'_{ij} \right) x_{ij} \right\} \to \min . \tag{4}$$

With this in mind, a mathematical model of the assignment task on the indicator of time costs can be presented in the following form:

$$\begin{cases} k_2(x) \to \min, \\ \sum\limits_{i=1}^{n} x_{ij} = 1,\ j = \overline{1,n};\ \ \sum\limits_{j=1}^{n} x_{ij} = 1,\ i = \overline{1,n};\ \ x_{ij} \in \{0,1\},\ i, j = \overline{1,n}. \end{cases} \tag{5}$$

In contrast to the traditional task to set the duration of the j-th channel of the i-th work, taking into account the waiting in the queue $\tau_{ij} = \left(\tau_{ij}^{o} + \tau'_{ij} \right)$, $i, j = \overline{1,n}$, it is necessary to take into account what work he executed in the previous cycle.

To assess the quality of execution of the work by the system $k_3(x) \to \max$, depending on the decision-making situation, the total or minimum value for the entire packet can be used:

$$k_3(x) = \sum_{i=1}^{n} \sum_{j=1}^{n} q_{ij} x_{ij} \to \max , \tag{6}$$

$$k_3(x) = \min_i \left\{ q_{ij} x_{ij} \right\} \to \max , \tag{7}$$

where q_{ij} quality of executing the i-th work by the j-th channel.

To assess options for the cyclic distribution of work simultaneously on the indicators of material (financial), time costs and quality of their implementation it is necessary to develop a multi-criteria task model.

Mathematical model of the multi-criteria work distribution task

In the cases when it is necessary to take into account several characteristics of variants of work distribution, as a rule, the transition to a single-objective task by convolution of the given local criteria is used. At that, the local criteria $k_l(x) \to \min(\max)$, $l = \overline{1,m}$ (where m is the number of local criteria of the task) can have different meaning, dimensionality, ranges, and directions of the desired change. To use local criteria in convolutions, their normalization is carried out. Due to the incomplete definiteness of decision-making situations for normalization the utility functions of local criteria $\xi_l(x)$, $l=\overline{1,m}$ are used, which are considered as membership functions of the fuzzy set «The Best Value». One of the most effective among them in terms of the complex indicator «accuracy-complexity» is the convolution function [12]:

$$\xi_l(x)=\begin{cases}\overline{a_l}\cdot\left(\dfrac{\bar{k}_l(x)}{\bar{k}_{al}}\right)^{\alpha_{1l}}, & 0\le\bar{k}_l(x)\le\bar{k}_{al};\\[2ex] \bar{a}_l+\left(1-\overline{a_l}\right)\left(\dfrac{\overline{k_l}(x)-\bar{k}_{al}}{1-\bar{k}_{al}}\right)^{\alpha_{2l}}, & \bar{k}_{al}<\overline{k_l}(x)\le 1,\end{cases} \tag{8}$$

$$\bar{k}_l(x)=\frac{k_l(x)-k_l^-}{k_l^+-k_l^-},\ l=\overline{1,m}, \tag{9}$$

where $\bar{k}_{al}$, $\overline{a_l}$ – coordinates of the glue point of the function, $0\le\bar{k}_{al}\le 1$, $0\le\overline{a_l}\le 1$; α_{1l}, α_{2l} – parameters that determine the appearance of the function on the initial and final segments; k_l^+, k_l^- – best and worst values of the local criterion $k_l(x)$.

Function (8) allows to realize linear and the most accurate non-linear in comparison with known functions (including *S*- and *Z*-shaped) approximation of the estimates of the local criterion values.

It is known that regardless of the direction of improvement of local criteria $k_l(x)\to\min$ or $k_l(x)\to\max$ their best values correspond to the maximum, and the worst - to the minimum value of the utility function $\xi_l(x)$, $l=\overline{1,m}$ (8). Taking this into account, as well as relations (8)–(9), the mathematical model of the multicriteria task of work distribution can be represented in the following form:

$$\begin{cases}\xi_1(x)\to\max,\\ \xi_2(x)\to\max,\\ \xi_3(x)\to\max,\\ \sum\limits_{i=1}^{n}x_{ij}=1\ \ \forall j=\overline{1,n},\ \ \sum\limits_{j=1}^{n}x_{ij}=1\ \ \forall i=\overline{1,n}.\end{cases} \tag{10}$$

Among the convolutions used in practice, the additive convolution, the method of the main criterion and the so-called guaranteed result method are most commonly used [9]. For the most complete accounting of the advantages of the decision maker, it is proposed to use the additive-multiplicative convolution, which is based on the Kolmogorov–Gabor function [13]:

$$P(x)=\sum_{l=1}^{3}\lambda_j\xi_j(x)+\sum_{l=1}^{3}\sum_{i=l}^{3}\lambda_{li}\xi_l(x)\xi_i(x)+\sum_{l=1}^{3}\sum_{i=l}^{3}\sum_{j=i}^{3}\lambda_{lij}(x)\xi_l(x)\xi_i(x)\xi_j(x), \quad (11)$$

where λ_l, λ_{li}, λ_{lij} – weighting coefficients of local criteria $k_l(x)$ and their products $\lambda_l \geq 0$, $\lambda_{li} \geq 0$, $\lambda_{lij} \geq 0$, $l,i,j=\overline{1,m}$.

The task of determining the parameters of the convolution function (11) can be solved by methods of ranking, hierarchy analysis, and sequential advantages. However, due to the complexity of expert evaluation technologies, it is proposed to use the comparator identification technology to solve the task of parametric synthesis of model (11) [13–15].

Determination of the costs of transition to the current work c_{ij}^{o}, $i,j=\overline{1,n}$ and the j-th channel delay time to complete the previous work τ_{ij}^{o}, $i,j=\overline{1,n}$ in the target functions (2)–(4) can be carried out using analytical or simulation modeling tools of mass service systems [11].

Conclusions

The solution to the urgent scientific and applied task of increasing the efficiency of work distribution technologies by developing mathematical models of cyclic distribution of work packages according to a set of indicators, taking into account the workload of executers has been obtained. The technology of work distribution and performance as a process of three-phase multi-channel mass service system is proposed. The discipline of service requests in such a system is determined by solving the task of work distribution between the channels, and the results of modeling cycles of its functioning allow us to determine the employment of channels by the time of distribution of the current work package.

Mathematical models of single-criteria and multi-criteria task with target functions of financial (material), time costs and quality of work are proposed. The parameters of the proposed models allow taking into account the employment of the channels at the time of distribution of the current work package and the costs of their readjustment after the previous work. For the most complete accounting of benefits in multi-criteria solutions, it is proposed to use an effective in terms of accuracy-complexity utility function of local criteria and universal additive-multiplicative convolution of criteria based on the Kolmogorov-Gabor function.

Partial variants of the model of the tricriteria task are double-criteria tasks with different combinations of local criteria. Parametric synthesis of models of double-criteria and tricriteria tasks can be carried out with the use of expert evaluation or comparator identification technologies.

The practical use of the proposed models will make it possible in practice to obtain the more effective solutions to the task of their distribution by taking into account the employment of channels and the costs of their readjustment after the previous work. The direction for further research may be the development of effective methods for modeling the processes of work execution and their implementation in the methods of work package distribution.

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