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STRUCTURAL AND PARAMETRICAL SYNTHESIS OF THE LAWS OF CRITICAL CONTROL

Timofeyev V., Khrustalova S., Khrustalev K., Yakushyk I.

Practical results related to the structural and parametric synthesis of critical control laws are presented. It is shown that when synthesizing digital controllers, it is most expedient to use discrete spaces that allow taking into account restrictions on the amplitudes of input signals. Approaches to the synthesis of critical controllers are considered. It is shown that the synthesis procedure is associated with minimizing the maximum absolute value of the generalized output for all possible values of input signals.

Introduction

In the practice of managing dynamic objects, one often has to deal with situations when the object does not belong to either statistical or stochastic ones, but is described by a set of target inequalities, that is, they are so-called critical [1].

For the synthesis of a critical control system, preliminary construction of both models of the control object itself and the environment is required. And if the object model can be described in terms of «Input – output», then the influence of the environment can be taken into account using a special description of the signals acting on the object [2–5].

The choice of one or another way of describing external signals is decisive in choosing a specific method for synthesizing the control law. At present, spaces $L(m,\delta)$ and $L(N,m_0,\delta_0)$ and are widely used, allowing to take into account restrictions on the amplitudes of input signals, and discrete spaces and, introducing restrictions on the space of the same signals.

General structure of the control law

Consider a discrete system $S_D(P,C)$ and a discrete space of inputs E . Let us set $\bar{w}=\left(y^{*},w\right)$ and write down the ratio

$$v(\bar{w},C): k \to v(k,\bar{w},C),\, k \in N\,, \tag{1}$$

which determines the relationship between the system output, external inputs and the structure of the control law C .

Then the efficiency of the functioning of such a system in the general case is determined by the criterion

$$J_E(C) = \sup\{|v(k,\bar{w},C)| : k \in N, \bar{w} \in E\}, \tag{2}$$

or, in the first approximation, the maximum absolute value of the generalized output for all possible inputs $\bar{w} \in E$ on the time interval $k \in N$.

In critical control systems, the main goal is to maintain a sufficiently low level of the output signal over the entire control interval, which can be expressed in the form of the inequality

$$J_E(C) \le \varepsilon_d , \tag{3}$$

where ε_d – is a positive value that determines the maximum possible value of $J_E(C)$.

At the same time, as noted above, the real control problem is described by a set of criteria specified in the form of a system of inequalities.

Consider the synthesis of a critical control law in the space of inputs $L(m_i, \delta_i)$, which ensures stable maintenance of the system of inequalities

$$J_L(C,\ m_i) \le \varepsilon_i,\ i = 0,1,2,\ldots,M , \tag{4}$$

where the number M specifies the number of all restrictions that must be maintained during the operation of the system. So, for example, if the input signal is $w(k) = 0.2(k+1)^{-1}\operatorname{sign}(\beta(k))$ for $k \le T_0$ and $w(k) = 0.1\cos k \times \operatorname{sign}(\alpha(k))$ for $k > T_0$, where $\alpha(k)$ and $\beta(k)$ – random variables with zero mathematical expectation, then the system of target inequalities can be specified in the form

$$\begin{cases} J_L(C,\infty) \le \varepsilon_1, \\ J_L(C,2) \le \varepsilon_2. \end{cases}$$

Note also that in a problem with one criterion $J_L(C,\infty) \le \varepsilon_1$ and inputs belonging to $L(\infty,1)$, the synthesis problem reduces to minimizing

$$\varepsilon_1 = \min_C \{J_L(C,\infty)\}$$

and is equivalent to an 1^1 optimization problem.

Let the control object be described by ARMAX, a model of the form

$$A(q)y(k) = q^{-d}B(q)u(k) + C(q)w(k), \tag{5}$$

where are polynomials $A \in R(q,n)$ with $a_0 = 1$, and $B \in R(q,m)$ and $C \in R(q,1)$ with $C_0 = 1$; $d \in N^+$; y, u and w are the generalized output, the control and external disturbing signals, respectively.

We also assume that the polynomials A and $q^{-d}B$ are coprime. Then the following equalities are true

$$AF + q^{-d}E = C, \tag{6}$$

$$AQ + q^{-d}BP = C, \tag{7}$$

where the polynomials $F \in R(q, d-1)$ with $f_0 = 1$, $E \in R(q, n-1)$, $Q \in R(q, d+m-1)$ with $q_0 = 1$, $P \in R(q, n-1)$, C are unique.

Using the parametrization introduced in [3], we can write the structure of the control law $C : y \to u$ in the form

$$C = (P + RA)\left(Q - Rq^{-d}B\right)^{-1}, \; R \in A_1, \tag{8}$$

where P and Q are determined by equation (7).

Using (5) and (8), we can determine the relationship between the output signal $y(k)$ and the external disturbing signal $w(k)$ in the form

$$y(k) = \left(Q - \bar{R}q^{-d}B\right)w(k), \tag{9}$$

and also, to estimate the value of the optimization criterion

$$J_L(C, m) = \left\| Q - Rq^{-d}B \right\|_{A_n} \delta, \tag{10}$$

where $n^{-1} + m^{-1} = 1$.

Estimate (10) can be obtained from the following considerations. We introduce a polynomial

$$H = Q - Rq^{-d}B$$

and write the expression following from (9)

$$y(k) = \sum_{i=0}^{k} h_i w(k-i). \tag{11}$$

Taking into account that $w \in L(m, \delta)$, and $n^{-1} + m^{-1} = 1$ and using Holder's inequality, we can write

$$J_L(C, m) \le \left\| Q - Rq^{-d}B \right\|_{A_n} \delta. \tag{12}$$

For each $k \in N^{+}$ there exists $w^{*} \in L(m, \delta)$, defined by the relation

$$w^{*}(k-i) = \begin{cases} \delta |h_i|^{n-1} \|H\|_{A_n}^{1-n} \operatorname{sign}(h_i), & if \quad 0 \le i \le k; \\ 0, \quad \text{otherwise}. \end{cases} \tag{13}$$

Substituting (13) into (11), we obtain

$$y^{*}(k)=\sum_{i=0}^{k}|h_i|^{n}\|H\|_{A_n}^{1-n}\delta$$

Since at $k\to\infty$

$$y(\infty)=\|H\|_{A_n}\delta,$$

then

$$J_L(C,m)\geq\left\|Q-Rq^{-d}B\right\|_{A_n}\delta. \quad (14)$$

Equation (10) follows from (12) and (14).

In the most general case, a polynomial under control actions can be represented as a product B_1B_2 , where the zeros B_1 lie outside the unit circle, and the zeros B_2 lie inside it. By setting $R=\bar{R}B_1$, we can introduce the ratio

$$Q-\bar{R}q^{-d}B=Q-Rq^{-d}B_2,$$

then, using (6) and (7), write down

$$Q=F+Dq^{-d},$$

where $D\in R(q,m-1)$.

So since

$$Q-\bar{R}q^{-d}B=F+\left(D-RB_2\right)q^{-d}$$

and

$$\left\|Q-\bar{R}q^{-d}B\right\|_{A_n}=\left(\|F\|_{A_n}^{n}+\|D-RB_2\|_{A_n}^{n}\right)^{1/n}, \quad (15)$$

it can be seen that the minimization problem $\left\|Q-\bar{R}q^{-d}B\right\|_{\bar{A}_n}$ can be reduced to minimization $\|D-RB_2\|_{\bar{A}_n}$, i.e.

$$\min_{\bar{R}\in A_1}\left\{\left\|Q-\bar{R}q^{-d}B\right\|_{A_n}\right\}\leftrightarrow\min_{R\in A_1}\left\{\|D-RB_2\|_{A_n}\right\}.$$

Thus, the characteristics of the control law are determined by the properties of the polynomial R, i.e. instead $J_L(C,m_i)$ of legal use $J_L(R,m_i)$. In this case, the problem of synthesizing the control law can be reformulated as follows: for given polynomials $D\in R(q,\bar{m}-1)$, $B_2\in R(q,r)$ and $d\in N^{+}$ find a polynomial R that ensures the fulfillment of the system of inequalities

$$J_L(R,m_i)\leq\varepsilon_i,\ i=0,1,...,M. \quad (16)$$

Denote by the R^0 polynomial $R \in A_1$ that ensures the fulfillment of (16). Then, as an admissible control law, one can use

$$C^0 = \left(PB_1 + R^0 A\right)\left(\left(Q - R^0 q^{-d} B_2\right) B_1\right)^{-1}, \quad (17)$$

in this case, in the general case, there may exist a set $R \in A_1$ satisfying (16).

Synthesis of the optimal controller

Let us consider the problem of optimizing the parameters of the control law (regulator) (17) in the proposal of the minimum-phase object (5) and show that the optimal controller can be written as

$$C^0 = E\left(BF\right)^{-1}, \quad (18)$$

while providing the minimum values of $J_L\left(C^0, m_i\right)$, i.e.

$$J_L\left(C^0, m_i\right) = \|F\|_{A_{n_i}} \ \delta_i, n_i^{-1} + m_i^{-1} = 1, \quad i = 1, 2, \ldots, M,$$

$$J_L\left(C^0, m_0\right) = \sum_{j=1}^{N} \|F\|_{A_{n_{0i}}} \ \delta_{0i}, \quad n_{0i}^{-1} + m_{0i}^{-1} = 1, \quad j = 1, 2, \ldots, N.$$

Due to the fact that the control object is the minimum phase, $B_1 = B$, and $B_2 = 1$. Then $R^0 = D$ provides a minimum $\| D - R\|_{A_{n_i}}$ because $\left\|D - R^0\right\|_{A_{n_i}} = 0$.

Substituting R^0 into (17) and using (6) and (7), we can obtain the control law (18). Since, then $R^0 = D$ from (15) it follows that

$$\min_{R \in A_1}\left\{\left\|Q - \bar{R} q^{-1} B\right\|_{A_{n_i}}\right\} = \left(\|F\|_{A_{n_i}}^{n_i} + \left\|D - R^0 B_2\right\|_{A_{n_i}}^{ni}\right)^{1/n_i} = \|F\|_{A_{n_i}}, \quad i = 1, 2, \ldots, M.$$

Let us further consider a special but fairly common case of the input space $L\left(\infty, \delta\right)$. From (15) it follows that

$$\min_{R \in A_1}\left\{\left\|Q - \bar{R} q^{-d} B\right\|_{A_1}\right\} = \|F\|_{A_1} + \min_{R \in A_1}\left\{\|D - RB_2\|_{A_1}\right\},$$

those only the member $\|D\text{-}RB_2\|_{A_1}$ is subject to minimization.

Assume further that B_2 contains the only zero inside the unit circle $q^{-1}=\sigma\left(|\sigma|\le 1\right)$. Then there exists $R\in R\left(q,m-2\right)$ minimizing $\|D\text{-}RB_2\|_{A_1}$, i.e.,

$$\min_{R\in R\left(q,m-2\right)}\left\{\|D-RB_2\|_{A_1}\right\}=\left|D\left(\sigma\right)\right|, \tag{19}$$

the optimal value of which is determined by the expression

$$R^0=\left(D\left(q^{-1}\right)-D\left(\sigma\right)\right)\left(q^{-1}-\sigma\right)^{-1}. \tag{20}$$

Since, $\|D-RB_2\|_\infty\le\|D-RB_2\|_{A_1}$ for any $R\in A_1$ it is obvious that

$$\min_{R\in A_1}\left\{\|D-RB_2\|_\infty\right\}\le\min_{R\in A_1}\left\{\|D-RB_2\|_{A_1}\right\}.$$

It is known [3, 4] that the choice R^0 according to (20) ensures the minimum of the left-hand side in (19), while $D-R^0B_2$ it acquires the value $D\left(\sigma\right)$, while it is obvious that and $\|D-RB_2\|_\infty$ and $\|D\text{-}RB_2\|_{\bar{A}_1}$ are also equal to $D\left(\sigma\right)$.

Next, consider the situation when the polynomial $D\in R\left(q,m-1\right)$, and the coefficients of the unstable polynomial $B_2\in R\left(q,r\right)$ satisfy the condition

$$\left|b_r\right|\ge\sum_{i=0}^{r-1}\left|b_i\right|. \tag{21}$$

If $k=m-r$, then from the expression

$$H=D-RB_2$$

should

$$\begin{cases} h_m=b_r r_{m-r}; \\ h_{m-1}=d_{m-1}-b_r r_{m-r-1}-b_{r-1}-b_{r-1}r_{m-r}; \\ h_{m-2}=d_{m-2}-b_2 r_{m-r-2}-b_{r-1}r_{m-r-1}-b_{r-2}r_{m-2}. \end{cases} \tag{22}$$

Introducing the estimate

$$J_k=\sum_{i=0}^{r+k}\left|h_i\right|,$$

from (22) one can obtain the inequality

$$J_{m-r}\ge J_{m-r-1}+\left(\left|b_r\right|-\sum_{i=0}^{r-1}\left|b_i\right|\right)\left|r_{m-r}\right|,$$

from which, as a result of minimization, in turn, follows

$$\min_{R\in R\left(q,m-r\right)}\left\{J_{m-r}\right\}\ge\min_{R\in R\left(q,m-r-1\right)}\left\{J_{m-r-1}\right\}+\min_{r_{m-r}\in R}\left\{\left(\left|b_r\right|-\sum_{i=0}^{r-1}\left|b_i\right|\right)\left|r_{m-r}\right|\right\}. \tag{23}$$

Then from (21) and (23) it is obvious that $r_{m-r}=0$, and

$$\min_{R\in R(q,m-r)}\left\{J_{m-r}\right\}\geq\min_{R\in R(q,m-r-1)}\left\{J_{m-r-1}\right\},$$

those the optimal value R belongs to $R(q,m-r-1)$.

Consider further the minimization problem $\|D-RB\|_{A_1}$ under the assumption that the polynomial B_2 has an arbitrary shape. Let us show that, in the general case, there are upper and lower bounds for the value of the minimum $\|D-RB_2\|_{A_1}$, i.e.,

$$\min_{R\in A_1}\left\{\|D-RB_2\|_\infty\right\}\leq\min_{R\in A_1}\left\{\|D-RB_2\|_{A_1}\right\}\leq\|D\|_{A_1}.$$

It follows from the definition that $\|\bullet\|_\infty$ and $\|\bullet\|_{A_1}$

$$\min_{R\in\bar{A}_1}\left\{\|D-RB_2\|_\infty\right\}\leq\min_{R\in\bar{A}_1}\left\{\|D-RB_2\|_{\bar{A}_1}\right\},$$

and since for $R=0$ we obtain the obvious inequality

$$D-RB_2=D_1$$

then

$$\min_{R\in A_1}\left\{\|D-RB_2\|_{A_1}\right\}\leq\|D\|_{A_1}.$$

This circumstance allows us to use the H^∞-optimization procedure to obtain the lower bound $\|D-RB_2\|_{A_1}$. This bound can be very useful for assessing the possibility of ensuring the fulfillment of the inequality $J_L(R,\infty)\leq\varepsilon_1$.

Let us assume that $D\in R(q,m-1)$, and $B_2\in R(q,r)$ has r different unstable roots. In this case, there is a finite $N_0\in N^+$ such that

$$\min_{R\in A_1}\left\{\|D-RB_2\|_{A_1}\right\}=\min_{R\in R(q,N_0)}\left\{\|D-RB_2\|_{A_1}\right\}.$$

Considering the polynomial $H=D-RB_2$ and minimizing the $\|H\|_{A_1}$ norm, we can use the well-known result, which says that there is a finite $M_0\in N^+$ such that the optimal polynomial satisfies the condition

$$H^0(\sigma_i)=D(\sigma_i),\ i=1,2,...r,$$

where σ_i are zeros B_2, i.e., $B(\sigma_i)=0$.

Since $H^0 - D$ the modulo is equal to $R^0 B_2$, then $R^0 \in R(q, M_0 - r)$. It can be seen that $R = R^0$ minimizes $\|D - RB_2\|_{A_1}$, while $N_0 = M_0 - r$.

Let us further consider a numerical method for finding the parameters of the synthesized controller that ensures the fulfillment of the system of inequalities (4).

Introducing the vector of coefficients of the polynomial $R \in A_1$ $r = (r_1, r_2, ...)^T$ and, replacing $J_L(R, m_i)$ by $J_L(r, m_i)$, we can rewrite the system of criteria (16) in the form

$$J_L(r, m_i) \le \varepsilon, \; i = 0,1,2,...,M. \qquad (24)$$

Denoting through S_i the set of all vectors of coefficients (controller parameters) that ensure the fulfillment of the i-th inequality

$$S_i = \{r : J_L(r, m_i) \le \varepsilon_i\},$$

one can write the intersection of the sets

$$S = \bigcap_{i=0}^{M} s_i, \qquad (25)$$

ensuring the fulfillment of the complete system (24).

To find the set (25), we will use the moving boundary method.

Let us first set $r \in R^j \left(j \in N^+ \right)$ and apply the method of moving boundaries to find the parameters of the control law. If the solution cannot be found for $r \in R^j$, then we assume $r \in R^{j+1}$ and repeat the search procedure again. The increase in order j continues until a solution is found that satisfies (24).

In some cases, the quality of the functioning of a critical system is determined mainly by only one of the criteria $J_L(r, m_i)$, $i = 0,1,2,\ldots,M$, which can be called the critical quality function. In this case, all other inequalities can be considered as constraints, which reduces the problem to the standard formulation adopted in optimization theory. To solve the problem, you can use both the considered method of moving boundaries and standard approaches adopted in non-linear programming.

We also note that although the standard methods 1^1-, H^2- and $H^\infty-$ optimization cannot be directly used to solve the problem of critical controller synthesis, they can be applied at the stage of preliminary estimation of the constraints that determine the structure and parameters of the system. So, if $\varepsilon_1 \le \inf\{J_L(r,1)\}$ or $\varepsilon_2 \le \inf\{J_L(r,2)\}$, with the help of these methods it is possible to evaluate whether such a controller exists at all. If the answer is negative, the problem should be reformulated already at the stage of formulation.

Synthesis of critical regulators

Let us consider the problem of synthesis of discrete critical algorithms in the spaces of inputs $D(m,\delta)$ or $D(N,m_0,\delta_0)$ according to the criterion

$$J_E(c) = \sup\left\{\left|v(k,\bar{w},c)\right| : k \in N, \bar{w} \in E\right\}, \tag{26}$$

associated with the maximum absolute value of the generalized output v for all input signals $\bar{w}$ in space E for all moments k of the control interval N [6]. In this case, as before, we will define the input space $D(m,\delta)$ as the set of all possible sequences w such that

$$\begin{cases} \sup\left\{\sum_{i=k}^{k+m}\left|\Delta w(i)\right| : k \in N \le \delta\right\}, \\ \left|w(k)\right| < \infty \quad \forall k \in N, \end{cases}$$

where $\delta \in (0,\infty)$, $m \in N^+$, $\Delta w(k) = w(k) - w(k-1)$, and the complex space $D(N,m_0,\delta_0)$ as the set of sequences w such that

$$w = \sum_{j=1}^{N} w^{(j)},$$

where $\left(w^{(1)}, w^{(\pi)}, \ldots, w^{(N)}\right) \in D(m_{01},\delta_{01}) \times D(m_{02},\delta_{02}) \times \ldots \times D(m_{0N},\delta_{0N})$.

In the general case, control algorithms related to criterion (26) are called critical ones [1], although there was no general approach to the synthesis of such procedures. A special case of critical control systems are critical regulators, in which there is no external setting signal.

Let us consider a system $S_D(P,C)$ in which there is no external setting signal y^*, i.e. the problem is reduced to stabilization of the output signal of the object in the vicinity of zero. In this case, the generalized output is replaced by the signal y, and the optimization criterion (26) takes the form

$$J_{DR}(C) = \sup\left\{\left|y(k,w,C)\right| : k \in N, w \in D\right\}, \tag{27}$$

where the symbol D denotes the input space $D(m,\delta)$ or $D(N,m_0,\delta_0)$.

The task of synthesis is to find the control law $C : y \to u$ that minimizes the objective function J_{DR}.

To simplify the calculations, we put $C(q)=1$ in ARMAX – models of the object included in the system $S_D(P,C)$, i.e.

$$A(q)y(k)=q^{-d}B(q)u(k)+w(k), \tag{28}$$

where is a polynomial $A(q)\in R[q,n]$ with, $a_0=1$, $B(q)\in R[q,m]$, pure delay time $d\in N^{+}$; y, u and w are the output, control, and disturbing signals, respectively.

Above, the validity of the identity

$$\Delta A(q)F(q)+q^{-d}E(q)=1, \tag{29}$$

in which the polynomials $F(q)\in R[q,d-1]$ with $f_0=1$ and $E(q)\in R[q,n]$ are unique.

Let us transform the description of the object (29) to the form of a d-step predictor, for which we multiply both parts of (29) by $\Delta F(q)q^{d}$

$$\Delta A(q)F(q)y(k+d)=\Delta F(q)B(q)u(k)+\Delta F(q)w(k+d), \tag{30}$$

after which, substituting identity (29) into (30), we obtain

$$\begin{cases} y(k+d)=\xi(k)+\Delta F(q)w(k+d), \\ \xi(k)=E(q)y(k)+\Delta F(q)B(q)u(k). \end{cases} \tag{31}$$

It follows from relations (31) that $y(k+d)$ it contains two terms: one of them is determined by known past control actions and measured outputs, and the other depends only on unchanging perturbations. Since the polynomial $F(q)$ has the order $d-1$, then all perturbations enter the description of the predictor with times greater than k, which in principle does not allow one to obtain estimates of the term from the measurement data $\Delta F(q)w(k+d)$.

Next, we return to the input space and consider the controller synthesis procedure for the case $0\le m<d-1$.

Set $\left|\Delta w(i)\right|=\gamma_i\delta$ for $i\in N$, where $\gamma_i\ge 0$. Then for $w\in D(m,\delta)$ the inequality

$$\sum_{i=k}^{k+m}\gamma_i\le 1 \quad \forall k\in N.$$

From (31) we further obtain the relations

$$\left|y(k+d)\right|\le\left|\xi_i\right|+\chi(\gamma_{k+1},\gamma_{k+2},...,\gamma_{k+d})\delta, \tag{32}$$

where

$$\chi(\gamma_{k+1},\gamma_{k+2},...,\gamma_{k+d})=\sum_{i=1}^{d-1}\left|f_i\right|\gamma_{k+d-i},$$

whence it follows that the synthesis problem can be reduced to finding a vector $(\gamma_{k+1},\gamma_{k+2},...,\gamma_{k+d})$ that maximizes the function χ or, what is the same, the objective linear function

$$\sum_{i=0}^{d-1}|f_i|\gamma_{k+d-i}$$

subject to restrictions

$$\gamma_{k+i}\ge 0,\quad i=1,2,...,d$$

and

$$\sum_{i=j}^{j+m}\gamma_i\le 1,\; j=k+1,k+2,...,k+d-m\,.$$

As can be seen, the synthesis problem is reduced to a standard linear programming problem, which can be solved in a finite number of steps using the simplex method. As a result of the decision, the optimal vector $\left(\gamma_{k+1}^0,\gamma_{k+2}^0,...,\gamma_{k+d}^0\right)$ is obtained, leading to the value of the objective function

$$\max\left\{\chi\left(\gamma_{k+1},\gamma_{k+2},...,\gamma_{k+d}\right)\right\}=\sum_{i=0}^{d-1}|f_i|\gamma_{k+d-i}^0\,.$$

It is easy to see that the formulation of the linear programming problem does not change for all $k\in N$, so the optimal solutions $\left(\gamma_1^0,\gamma_2^0,...,\gamma_d^0\right)$ for all times k are the same.

Let us denote these solutions in the form $(\gamma_1^0,\gamma_2^0,...,\gamma_d^0)$, whence

$$\max\left\{\chi\left(\gamma_{k+1},\gamma_{k+2},...,\gamma_{k+d}\right):k\in N\right\}=\sum_{i=0}^{d-1}|f_i|\gamma_{d-i}^0\,.$$

Then we can rewrite (32) as

$$\left|y(k+d)\right|\le\left|\xi(k)\right|+\sum_{i=0}^{d-1}|f_i|\gamma_{d-i}^0\delta\,,$$

leading to an assessment of the quality of regulation

$$J_{DR}(C)\le\sup\left\{\left|\xi(k)\right|:k\in N\right\}+\sum_{i=0}^{d-1}|f_i|\gamma_{d-i}^0\delta\,. \qquad (33)$$

A special case of perturbation for this situation is $w^*\in D(m,\delta)$, defined by the relation

$$w^*(k)=\begin{cases}0, & for\quad k\le 0,\\ w^*(k-1)+\delta\gamma_k^0\,\mathrm{sign}\left(f_{d-k}\right), & for\quad 0<k\le d,\\ w^*(k-1), & for\quad k>d\;.\end{cases}$$

Considering further the controller C_R in the form

$$\Delta F(q)B(q)u(k) = -E(q)y(k) \ , \tag{34}$$

one can see that $\xi(k) = 0$, and

$$y\left(d, w^*, C_R\right) = \sum_{i=0}^{d-1} |f_i| \gamma_{d-i}^0 \delta \, , \tag{35}$$

after which, using (33) and (35), we can conclude that the criterion value $J_{DR}(C)$ cannot be less than $y\left(d, w^*, C_R\right)$.

Thus, any controller that provides the value of the objective function

$$J_{DR}(C_R) = \sum_{i=0}^{d-1} |f_i| \gamma_{d-i}^0 \delta$$

and the value of the output signal

$$y(k, w, C_R) = \Delta F(q) w(k)$$

is critical.

In the particular case for $m = 0$, it is easy to see that the vector of optimal solutions $\left(\gamma_1^0, \gamma_2^0, ..., \gamma_d^0\right)$ is $(1,1,\ldots,1)$, and the target solution takes the form

$$\sum_{i=0}^{d-1} |f_i| \gamma_{d-i}^0$$

under restrictions

$$0 \le \gamma_i \le 1, \quad i = 1, 2, ..., d \, .$$

Obviously, the optimal value of the criterion for $m = 0$ is

$$J_{DR}(C_R) = \sum_{i=0}^{d-1} |f_i| \delta \, .$$

A more complicated situation arises in the case of $m \ge d - 1$. From (31) it follows that

$$\left|y((k+d)\right| \le \left|\xi(k)\right| + |f_0| \left|\Delta w(k+d)\right| + |f_1| \left|\Delta w(k+d-1)\right| + ...$$

$$+ |f_{d-1}| \left|\Delta w(k+1)\right| + \left\|F(q)\right\|_{A\infty} \sum_{i=k+1}^{k+d} \left|\Delta w(i)\right|,$$

or

$$y(k+d) \le \left|\xi(k)\right| + \left\|F(q)\right\|_{A\infty} \delta \, .$$

Taking into account (27), this inequality leads to an estimate of the control quality

$$J_{DR}(C) \le \sup\left\{\left|\xi(k)\right| : k \in N\right\} + \left\|F(q)\right\|_{A\infty} \delta \, .$$

For perturbations $w^* \in D(m,\delta)$ of the form

$$\Delta w^*(k) = \begin{cases} \delta\, sign(f_M), & if \quad k = M, \\ 0, & if \quad k \neq M, \end{cases}$$

where f_M – is the largest of the values $\{f_0, f_1, ..., f_{d-1}\}$, controller (34) leads to $\xi(k) = 0$ and the estimate

$$y\left(d, w^*, C_R\right) = \delta |f_M| = \|F(q)\|_{A\infty} \delta \ ,$$

those is also critical.

Thus, it can be argued that the critical controller C_R is invariant to the parameters m and δ the input space $D(m,\delta)$.

For the input space $D = D(N, m_0, \delta_0)$ and perturbations $w \in D(N, m_0, \delta_0)$, controller (34) is also critical. This can be shown by introducing the function

$$Q_R(k) = \begin{cases} \sum_{i=0}^{d-1} |f_i| \gamma_{d-i}^0, & 0 \leq k < d-1, \\ \|F(q)\|_{A\infty}, & k \geq d-1, \end{cases}$$

after which, carrying out transformations similar to the previous one, we obtain

$$J_{DR}(C_R) = \sum_{i=1}^{N} Q_R(m_{0i}) \delta_{0i} \ ,$$

$$y(k, w, C_R) = F(q) \sum_{i=1}^{N} \Delta w^{(i)}(k),$$

where the polynomial is defined by relation (29).

Thus, the introduced supremal controller ensures the quality of control

$$J_{DR}(C_R) = \begin{cases} Q_R(m)\delta, & for \quad D = D(m,\delta), \\ \sum_{i=1}^{N} Q_R(m_{0i})\delta_{0i}, & for \quad D = D(N, m_0, \delta_0). \end{cases}$$

Conclusion

Practical results related to the structural and parametric synthesis of critical control laws in various metric spaces are presented. The general structure of the critical control law is proposed. Approaches to the synthesis of critical controllers are considered. A procedure for the synthesis of critical controllers based on multi-step optimal predictors has been introduced and justified. A procedure for calculating the parameters of an optimal controller based on the solution of a standard linear programming problem is proposed, and the optimality of the resulting solution is proved. The stability of a closed critical control system in the case of non-stationary external disturbances is analyzed. It is shown that after the end of the transient processes, the system «contracts» into a tube, the characteristics of which are determined both by the properties of the object itself (location of zeros and poles) and of the acting disturbances.

References

1. Zakian V., Al-Naib V. Design of Dynamical and control systems by the method of inequalities // Proc. IEE. – 1973. – 120. – №11. – P. 1421–1427.
2. Doyle J. C., Francis B. A., Tannenbaum A. R. Feedback Control Theory. – N. Y.: Macmillan, 1991. – 362 p.
3. Clarke D. W., Mohtadi C., Tuffs P. S. Generalized predictive control. Tue basis algorithm // Automatica. – 1987. – 22. – №2. – P. 137–148.
4. Parlos A. G., Henry A. F., Schweppe F. C., Gould J. A, Janning D. D. Nonlinear multivariable control of nuclear power plants based on the unknown – but – bounded disturbance model // IEEE Trans. on Custom. Control. – 1988. – 33. – P. 130–137.
5. Bakan G.M., Volosov V.V., Kussul N.N. Estimation of the state of continuous dynamical systems by the method of ellipsoids // Cybernetics and system analysis, 1996. – №6. – P. 72–91.
6. Whidlorue J. F., Jiu G. P. Critical Control Systems. – N. Y.: John Wiley & Sons Inc., 1993. –187 p.
7. Timofeyev V.A., Ilyunin O.O., Samer Laga. Synthesis of a modified critical controller for the control of non-stationary objects // Bulletin of the Kherson National Technical University. – Kherson, No. 38. – 2010. – P. 398–401.