# **SIMULATION MODELLING OF THE PROCESS OF DISTRIBUTION AND EXECUTION OF WORK PACKAGES**

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*A new solution to the scientific and applied problem of increasing the efficiency of technological systems has been obtained. A model of the problem of distribution of work packages by indicators of material (financial) costs, system productivity and quality of work performance by elements with different functional and cost characteristics is proposed. Considering the probabilistic nature of the input flow of work packages and the time of their execution, it is proposed to determine the system performance using simulation statistical modelling of the system as a three-phase multi-channel queuing system. The software implementation of the model is carried out using a package of simulation statistical modelling software. Solutions to the problems of tactical planning of computer experiments are proposed, which allow obtaining results of the required accuracy and reliability. The developed models help to determine the characteristics of technological systems with an unequal number of elements and different laws of distribution of packet flow and time of execution of various works. The practical application of the proposed models will allow, in practice, taking into account the occupancy of channels and the cost of their reconfiguration, to obtain more efficient solutions to the problems of their distribution after the preliminary work has been completed.* 

### **Introduction**

Rapid changes in the conditions of operation of production, technological, information, service, and other types of systems, in particular in a pandemic and martial law, necessitate their reengineering. Reengineering projects for such systems involve solving a set of multi-criteria problems of their structural, parametric, topological, and functional optimisation in conditions of incomplete certainty. The reengineering of modern facilities is carried out using a systematic approach that involves its decomposition into packages (complexes) of work and individual works. At the same time, in the management of reengineering projects and i n the management of such systems, there is a need to optimise the distribution of work between their elements (companies, departments, individual executors, equipment, etc.) under certain restrictions on quality, material, time, financial costs, or profit  $[1-4]$ .

In many cases, work allocation problems can be reduced to a classic assignment problem or assignment problems with additional requirements. Examples include the works of allocating construction work, developing software systems, manufacturing, and repair work, etc. [5].

The mathematical model of the classical assignment problem with the number of works equal to the number of executors in terms of costs can be<br>presented as follows [2]:<br> $\int f(x) = \sum_{i=1}^{n} \sum_{j=1}^{n} a_{ij} x_{ij} \rightarrow \min_{x}$ presented as follows [2]:  $\sum_{n=1}^{n} \sum_{i=1}^{n} a_{ii}x_{ii}$ 

equal to the number of executors in terms of costs can be  
\ns [2]:  
\n
$$
\begin{cases}\nf(x) = \sum_{i=1}^{n} \sum_{j=1}^{n} a_{ij} x_{ij} \rightarrow \min_{x}, \\
a_{ij} > 0, \ i, j = \overline{1, n}, \\
\sum_{i=1}^{n} x_{ij} = 1, \ j = \overline{1, n}; \\
\sum_{j=1}^{n} x_{ij} = 1, \ i = \overline{1, n}; \\
x_{ij} \in \{0, 1\}, \ i, j = \overline{1, n},\n\end{cases}
$$
\n(1)

where *n*- number of works and executors;  $a_{ij}$  – costs (time, material, financial) for the performance of the *i*-th work by the *j*-th executor;  $x = \left[ x_{ij} \right], i, j = \overline{1,n}$  – assignment matrix (element  $x_{ij} = 1$ , if the work *i* is assigned to the executor *j*;  $x_{ij} = 0$  – otherwise).

The features of some systems as objects of reengineering or management do not meet the requirements of the model of the classical problem  $(1)$  [5–10]: the goal may be to find the maximum of the objective function (profit, quality of work, etc.)  $f(x) \rightarrow \max$ ; there may be several objective functions; the number of works *n* may not be equal to the number of executors  $r$ ; there may be prohibitions on the assignment of certain works to certain executors; material (financial), time costs and other process parameters may be non-deterministic.

Taking into account the dynamics of the input flow of work packages, incomplete certainty of the parameters of the object functioning process, and the qualifications of executors, there is a need to jointly solve the problems of determining the workload of executors and work distribution. Therefore, the scientific and applied task of increasing the efficiency of technological systems by developing an analytical and simulation model of the cyclic distribution of work packages by a set of indicators, taking into account the workload of performers and incomplete certainty of process parameters, is relevant.

*The object of the study* is technological systems designed to perform design, construction, repair, and other types of work packages under conditions of incomplete certainty.

*The subject of the study* is the processes of cyclical distribution of work packages by a set of indicators and their implementation, taking into account the qualifications and workload of performers.

*The purpose of the study* is to develop an analytical and simulation model of cyclic distribution by a set of indicators and execution of work packages under conditions of incomplete certainty to improve the efficiency of technological systems.

#### **Statement of the problem**

A system where the technology for executing works packages consists of three phases is considered. Work packages are received at the system input at random moments of time. In the first phase, the supervisor distributes work packages  $n = var$ of different specializations among  $r \geq n$  performers of different qualifications according to the indicators of costs, efficiency and quality. The material (financial) costs, time and quality of each specialization depend on the qualifications of the performer and are random variables with specified distribution laws. The third phase involves the aggregation (compilation, quality assessment, documentation, etc.) of the completed package works. Its duration is a random variable with a given distribution law.

It is necessary to develop a mathematical model of the processes of cyclic distribution and execution of work packages, which will allow determining the indicators of costs, productivity, and quality of the completed work for a given structure and parameters of the system elements.

## **A mathematical model of a multi-criteria work package distribution task**

The efficiency of a technological system is determined by the ratio of the effect obtained from its use to the costs of obtaining it, taking into account the quality of work execution. We formalize the task of assigning  $r$  system executors to execute the *n* work package according to the following indicators: material (financial) costs  $k_1(x) \to \min$ , time for work execution  $k_2(x) \to \min$ , which determines the system productivity, and quality of execution of the entire work package  $k_3(x) \rightarrow \max$ .

After performing the previous transformations, you can get a square matrix  $a = \lfloor a_{ij} \rfloor$ ,  $i, j = \overline{1,n}$  and go to the traditional task where the number of executors is equal to the number of works  $r = n$  (1). Then, for example, the Hungarian method can be used to solve it.

Target function of material (financial) costs of the work package in the second phase is:

$$
k_1(x) = \sum_{i=1}^{n} \sum_{j=1}^{n} c_{ij} x_{ij} \to \min_{x} ,
$$
 (2)

where  $c_{ij} = \left(c_{ij}^o + c_{ij}^r\right)$ ,  $i, j = \overline{1, n}$  – general material (financial) costs of execution of the *i*-th work by the *j*-th executor;  $c_{ij}^o$  – costs of moving to the execution of the current work after the execution of the work of the previous package;  $c'_{ij}$  – nominal costs of executing of the  $i$ -th work by the  $j$ -th executor.

Taking into account the parallel independent execution of works, it is proposed to choose the time of execution of the maximum duration of the work as the indicator of efficiency of the second phase:

and phase:

\n
$$
k_2(x) = \max_{i} \left\{ \tau_{ij} x_{ij} \right\} \to \min_{x},\tag{3}
$$

where  $\tau_{ij} = \left(\tau_{ij}^0 + \tau_{ij}^1\right)$ ,  $i, j = \overline{1,n}$  – total expenditure of execution time of *i*-th work by the *j*-th executor;  $\tau_{ii}^0$  $\tau_{ij}^0$  – time to transition to the current work after completing the work of the previous package;  $\tau'_{ij}$  – nominal execution time for the *i*-th workby the  $j$ -th executor.

To assess the quality of the works in the second phase, it is proposed to use its minimum value among all works of the package:

rks of the package:  
\n
$$
k_3(x) = \min_i \{q_{ij}x_{ij}\} \rightarrow \max_i,
$$
\n(4)

where  $q_{ij}$  – quality of the execution of the *i*-th work by the *j*-th executor.

To evaluate the options for the distribution and execution of works simultaneously in terms of material (financial), time costs and quality of their execution (2)–(4), we can use different convolutions of normalised values of local criteria. Due to the incomplete certainty of the situations of works distribution, we will use utility functions to normalise the local criteria  $\xi_l(x)$ ,  $l = 1,3$ , which are considered as membership functions of the fuzzy set "best value". Let's use the classical utility function, which is a special case of the universal glue function [11]:

$$
\xi_l(x) = \left[\frac{k_l(x) - k_l^-}{k_l^+ - k_l^-}\right]^{\alpha_l}, \ l = \overline{1,3}, \tag{5}
$$

where  $k_l^+$ ,  $k_l^-$  – best and worst values of the local criterion  $k_l(x)$ ;  $\alpha_l$  – parameters that determine the type of function (convex, linear, or concave).

Regardless of the direction of improvement of local criteria values  $k_l(x) \to \min$  (2)–(3) or  $k_l(x) \to \max$  (4), their best values correspond to the maximum and worst values to the minimum values of the utility function  $\xi_l(x)$ ,  $l = 1,3$  (5) in the range from 0 to 1.

Taking into account that the total costs of the first and third phases of material (financial) resources  $c_{\Delta}$  and time  $\tau_{\Delta}$  do not depend on the distribution of works for the second phase, the general objective functions for them can be presented as follows:

$$
k_1(x) = c_{\Delta} + \sum_{i=1}^{n} \sum_{j=1}^{n} c_{ij} x_{ij} \rightarrow \min_{x},
$$
 (6)

$$
k_1(x) = c_{\Delta} + \sum_{i=1}^{n} \sum_{j=1}^{n} c_{ij} x_{ij} \rightarrow \min_{x}
$$
  
\n
$$
k_2(x) = \tau_{\Delta} + \max_{i} \{\tau_{ij} x_{ij}\} \rightarrow \min_{x}
$$
 (7)

Using the convolution of utility functions  $(5)$  of local criteria  $(6)$ – $(7)$ , the mathematical model of the multicriteria problem of distribution and execution<br>of work packages is proposed to be presented in the following form:<br> $P(x) = \sum_{l=1}^{3} \lambda_l \xi_l(x) \rightarrow \max_{x}$ ,

of work packages is proposed to be presented in the following form:  
\n
$$
\begin{cases}\nP(x) = \sum_{l=1}^{3} \lambda_l \xi_l(x) \rightarrow \max, \\
\sum_{i=1}^{n} x_{ij} = 1, j = \overline{1, n}; \sum_{j=1}^{n} x_{ij} = 1, i = \overline{1, n}; x_{ij} \in \{0, 1\}, i, j = \overline{1, n},\n\end{cases}
$$
\n(8)

where  $\lambda_l$  – weighting coefficients of local criteria,  $\lambda_l \geq 0$ ,  $l = 1,3$ , 3 1  $l = 1$ *l*  $\lambda$ i  $=$  $\sum \lambda_l = 1$ .

If there is additional information about the decision maker's preferences, an additive-multiplicative convolution based on the Kolmogorov–Gabor polynomial If there is additional information about the decision maker's preferences,<br>additive-multiplicative convolution based on the Kolmogorov–Gabor polynomial<br>be used as an objective function [12–13]:<br> $(x) = \sum_{l=1}^{3} \lambda_j \xi_j(x) + \sum_{l$ If there is additional information about the decision maker's preferences,<br>
additive-multiplicative convolution based on the Kolmogorov-Gabor polynomial<br>
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can be used as an objective function [12–13]:  
\n
$$
P(x) = \sum_{l=1}^{3} \lambda_j \xi_j(x) + \sum_{l=1}^{3} \sum_{i=l}^{3} \lambda_{li} \xi_l(x) \xi_i(x) + \sum_{l=1}^{3} \sum_{i=l}^{3} \sum_{j=i}^{3} \lambda_{lij}(x) \xi_l(x) \xi_j(x) \xi_j(x),
$$
\n(9)  
\nwhere  $\lambda_l$ ,  $\lambda_{li}$ ,  $\lambda_{lij}$  – weighting coefficients of local criteria and their products

 $\lambda_l \geq 0$ ,  $\lambda_{li} \geq 0$ ,  $\lambda_{li} \geq 0$ ,  $l, i, j = \overline{1,3}$ .

The task of determining the parameters of the convolution functions (8)–(9) can be solved by the methods of ranking, hierarchy analysis, sequential preferences, or comparative identification [12–13].

Determination of the quality of performance and the cost of material (financial) resources for the objective function in (8) for a given distribution of works can be carried out directly using relations (4) and (6).

If the parameters  $c_{\Delta}$ ,  $\tau_{\Delta}$ ,  $c_{ij}$ ,  $q_{ij}$ ,  $i, j = 1, n$  in the proposed model will be given in the form of intervals, it is proposed to use the interval representation of the characteristics of the options for the distribution of works  $x \in X$  (where X is the set of valid distribution options). In this case, each of the characteristics will be represented not by one value, but by two, which will define its boundaries  $k_j(x) = [k_j^-(x); k_j^+(x)], \ j = \overline{1,3}.$ 

For interval values  $a \in |a^{\text{-}}; a^+|$  $\left\lfloor a^{-}; a^{+}\right\rfloor$  and  $b \in b^{-}; b^{+}\right\rfloor$ o  $\left[ b^{-};b^{+}\right]$  of local criteria  $k_{j}(x)$ ,

 $j = 1,3$  the rules for performing classical arithmetic operations are determined by the relations [14]:

$$
\left[c^{-};c^{+}\right] = \left[a^{-};a^{+}\right] \circ \left[b^{-};b^{+}\right];\tag{10}
$$

$$
\begin{bmatrix} a \\ \end{bmatrix} + \begin{bmatrix} b \\ \end{bmatrix} = \begin{bmatrix} a^- + b^-; a^+ + b^+ \end{bmatrix};\tag{11}
$$

$$
[a] - [b] = \begin{bmatrix} a^- - b^+; a^+ - b^- \end{bmatrix};\tag{12}
$$

$$
[c^-,c^+] = [a^-,a^+] \circ [b^-,b^+]; \qquad (10)
$$

$$
[a]+[b] = [a^-+b^-,a^++b^+]; \qquad (11)
$$

$$
[a]-[b] = [a^- - b^+,a^+-b^-]; \qquad (12)
$$

$$
[a]\cdot [b] = [\min\{a^-b^-,a^-b^+,a^+b^-,a^+b^+ \}; \max\{a^-b^-,a^-b^+,a^+b^-,a^+b^+ \}]; \qquad (13)
$$

$$
[a]/[b] = [a]\cdot [1/b^+;1/b^-], \ b^- \neq 0, \ b^+ \neq 0 \ . \qquad (14)
$$

$$
[a]/[b] = [a] \cdot [1/b^+; 1/b^-], b^- \neq 0, b^+ \neq 0.
$$
 (14)

Comparison of estimates of distribution options represented by nonoverlapping intervals will be carried out by comparing their centres (mean values). For interval values of local criteria that overlap, it is proposed to use the estimate of the generalised Hukuhara difference (interval difference, *gH*-difference) [15, 16].

In this case, we will represent the values of the  $j$ -th characteristic of the works distribution options  $x_i, x_i \in X$  as intervals  $A = \left[ k_j^-(x_i) ; k_j^+(x_i) \right]$  and  $B = \left[ k_j^-(x_l); k_j^+(x_l) \right]$  in the form  $A = \left[ \hat{a}; \overline{a} \right]$  and  $B = \left[ \hat{b}; \overline{b} \right]$  where  $\hat{a}, \hat{b}, \overline{a}, \overline{b}$  are the centres and radii of the intervals  $A$  and  $B$ , respectively:

$$
\hat{a} = \left[a^- + a^+\right] \Big/ 2 \,, \ \overline{a} = \left[a^+ - a^-\right] \Big/ 2 \,, \tag{15}
$$

$$
\hat{b} = \left[b^- + b^+\right] \Big/ 2, \ \overline{b} = \left[b^+ - b^-\right] \Big/ 2. \tag{16}
$$

Then the generalised Hukuhara difference  $A_{gH}^- B$  and the comparison index  $\gamma_{A,B}$  built on its basis for the entered intervals  $A = [\hat{a}; \overline{a}]$  and  $B = [\hat{b}; \overline{b}]$ will be determined by the following relations [15, 16]: generalised Hukuhara difference  $A^{-}_{gH}B$  and the comp<br>
i on its basis for the entered intervals  $A = [\hat{a}; \overline{a}]$  and  $B = [\overline{a}$ <br>
ed by the following relations [15, 16]:<br>  $\left\{a^{-} - b^{-} ; a^{+} - b^{+}\right\}$ ; max  $\left\{a^{-} - b^{-} ; a^{+} - b$ dex  $\gamma_{A,B}$  built on its basis for the entered intervals  $A = [\hat{a}; \overline{a}]$  and  $B = [\hat{b}; \overline{b}]$ <br>ill be determined by the following relations [15, 16]:<br> $A_{gH}^- B = [\min \{a^- - b^-; a^+ - b^+\}; \max \{a^- - b^-; a^+ - b^+\}]= (\hat{a} - \hat{b}; |\overline{a} - \overline{b}|),$ 

$$
A_{gH}^{-}B = \left[\min\left\{a^{-} - b^{-}; a^{+} - b^{+}\right\}; \max\left\{a^{-} - b^{-}; a^{+} - b^{+}\right\}\right] = \left(\hat{a} - \hat{b}; \left|\bar{a} - \bar{b}\right|\right), (17)
$$

$$
\gamma_{A,B} = \left(\bar{a} - \bar{b}\right) / \left(\hat{a} - \hat{b}\right). \tag{18}
$$

$$
\gamma_{A,B} = \left(\overline{a} - \overline{b}\right) / \left(\hat{a} - \hat{b}\right). \tag{18}
$$

In this case, the comparison index  $\gamma_{A,B}$  (18) will have the meaning of a measure of gain or risk, when the interval A is chosen instead of B just on the basis of fulfilling the inequality  $\hat{a} > \hat{b}$ .

Taking into account the probabilistic nature of the input flow of work packages and the time of their execution in the second phase, it is proposed to calculate the values of the objective function (7) using simulation modelling of the process as a queuing system. This involves the development of a modelling algorithm for the general process of distribution and execution of works.

### **Modelling algorithm of the system operation process**

Based on the stochasticity of the flow of work packages and the duration of their execution, it is proposed to consider such systems as three-phase multichannel queuing systems (*Q*-schemes) (Fig. 1):

$$
Q = \langle W, U, H, Z, R, A \rangle, \tag{19}
$$

where  $W$  – input flow of applications (work packages);  $U$  – service flow (distribution laws and parameters for the time of work execution by the channels of each phase); *H* – set of internal parameters (number of channels of the second phase, permissible length of queues in front of them); *Z* – set of permissible system states (occupancy of queues and channels);  $R$  – scheme of connections between system elements; *A* – system functioning algorithm (method of work distribution, service discipline)

The source (generator)  $G$  in the  $Q$ -scheme (Fig. 1) generates a stream of offers (work packages) that enter the system at random moments in time. The channel  $C_{1,1}$  simulates the process of distributing orders (works packages), which are sent through the valve system to be executed by the channels of the second  $C_{2,1}$ ,  $C_{2,2}$ , ...,  $C_{2,n}$  and third  $C_{3,1}$  phases. Before the channels of each

of the phases, there may be corresponding queues of orders  $Q_{1,1}$ ,  $Q_{2,1}$ ,  $Q_{2,2}$ , ...,  $Q_{2,n}$  and  $Q_{3,1}$ .



**Fig. 1.** Representation of the system in *Q*-chart notation

The purpose of modelling the system is to determine the total time The purpose of modelling the system is to determine the of distribution and execution of orders  $\tau(x) = \tau_{\Delta} + \max_i \{\tau_{ij}x_{ij}\}$  (7). T  $\tau(x) = \tau_{\Delta} + \max\left\{\tau_{ij}x_{ij}\right\}$  (7). To solve the problem, it is proposed to use the methodology of analytical and simulation statistical modelling [17]. Modelling algorithms based on the event principle are more accurate and economical for studying such objects. Taking into account the peculiarities of the system, a modelling algorithm based on the principle of sequential applications, which is a modification of the event principle for the study of queuing systems, is proposed to solve the problem of determining the indicators of costs, productivity and quality of work performed.

The proposed modelling algorithm involves the implementation of the following steps.

*Start*. Input of information (types and parameters of the laws of distribution of the flow of requests, time  $\lfloor \tau_{ij} \rfloor$  and quality  $\lfloor q_{ij} \rfloor$  of execution of requests, resource  $\left[ c_{ij} \right]$  consumption by channels of each phase, weighting coefficients and parameters of utility functions of local criteria  $[\lambda_l]$ , initial values  $[p_{ij}(x)]$ ,  $i, j = \overline{1,n}$  for the distribution of requests of the first batch, formation of initial conditions (values of the time counter, number of received requests (batches), variables for collecting statistics, state of queues and channels of each phase, etc.), determination of conditions for stopping the simulation.

*Step 1*. Based on the information about the type and parameters of the flow distribution law, generate the moment of receipt of the next order (work package).

*Step 2.* Modelling the process of an application's stay in the first phase (taking a place in the queue  $Q_{1,1}$ ; occupying a channel  $C_{1,1}$ ; releasing a place in the queue  $Q_{1,1}$ ; if the order is not the first, calculating the value of the total utility of the distribution of works among the channels of the second phase  $[p_{ij}(x)]$ ,  $i, j = \overline{1,n}$  using the relations (8) or (9); allocating a family of orders for execution by the channels of the second phase using the Hungarian algorithm (8)  $x = \lfloor x_{ij} \rfloor$ ,  $i, j = \overline{1, n}$ ; releasing a channel  $C_{1,1}$ ).

*Step 3*. Modelling the process of staying of a family of orders in the second phase (taking places in queues  $Q_{2,1}$ ,  $Q_{2,2}$ , ...,  $Q_{2,n}$ ; occupation of channels  $C_{2,1}$ ,  $C_{2,2}, \ldots, C_{2,n}$ ; release of places in queues  $Q_{2,1}, Q_{2,2}, \ldots, Q_{2,n}$ ; servicing of orders; release of channels  $C_{2,1}$ ,  $C_{2,2}$ , ...,  $C_{2,n}$ ; combining a family of orders).

*Step 4.* Modelling the process of an order being in the third stage (taking a place in the queue  $Q_{3,1}$ ; channel occupation  $C_{3,1}$ ; release of space in the queue  $Q_{3,1}$ ; servicing of the order; release of the channel  $C_{3,1}$ ).

*Step 5*. Calculation of the values of local quality criteria for the processed order (work package)  $k_3(x) = \min_i \left\{ q_{ij} x_{ij} \right\}$  (4),  $k_3(x) = \min \{ q_{ij} x_{ij} \}$  (4), costs  $k_1(x) = c_{\Delta}$ 1 1 *n n*  $i_{ij}x_{ij}$  (  $\begin{array}{c} 2 \leq t \\ i=1 \end{array}$ <br>*i*=1  $k_1(x) = c_{\Delta} + \sum_{i=1}^{n} \sum_{j=1}^{n} c_{ij} x_{ij}$  (  $\sum$   $\sum$   $\sum$   $\sum$   $i$  $=c_{\Delta} + \sum_{i=1}^{n} \sum_{j=1}^{n} c_{ij} x_{ij}$  (6) and service time of the order  $k_2(x) = \tau_{\Delta} + \max_i \{\tau_{ij}x_{ij}\}$  (7).

 $k_2(x) = \tau_{\Delta} + \max\{ \tau_{ij} x_{ij} \}$  (7).

*Step 6.* Statistics collection on the time of servicing requests, queue status, channels, etc.

*Step 7*. If the conditions for stopping the algorithm are not met, then the algorithm proceeds to Step 1, otherwise – to Step 8.

*Step 8*. Statistical and analytical processing of modelling results (determination of the number of orders processed, assessment of cost, productivity and quality of work performed, characteristics of queues and channels of the system, etc.)

*Completion*. Output of modelling results.

The software implementation of the developed modelling algorithm and experiments make it possible to obtain the necessary results, the accuracy of which will increase with the number of experiments. To obtain the results of a given accuracy using the developed model under conditions of limited time and computing resources, it is necessary to solve the tasks of tactical planning of machine

experiments: selection of initial conditions of modelling; assessment of the accuracy of modelling results and selection of the required number of experiments; reduction of the variance of the obtained estimates; determination of conditions for stopping modelling experiments.

In the process of selecting the initial conditions of modelling, taking into account the intensity of the flow of requests and the time of their service by channels, it is proposed to use analytical relations that allow an approximate assessment of the channel load and the length of the respective queues. The values obtained in this way are proposed to be selected as initial conditions.

In the process of evaluating the accuracy of the results, we will use its average value 1 1 *N*  $\sum_{i=1}^{n} z_i$  $\overline{\tau} = \frac{1}{\sqrt{2}} \sum \tau_i$  $=$  $=\frac{1}{N}\sum \tau_i$  based on the simulation results as an estimate of the time of work package  $\tau(x)$  distribution and execution (where  $\tau_i$  – is the time of work package distribution and execution in the  $i$ -th experiment;  $N -$  is the number of experiments). The accuracy (error) of the estimate will be determined by the ratio:  $\varepsilon = |\tau(x) - \overline{\tau}|$ . The probability  $\alpha$  that the obtained error value  $\varepsilon$ does not exceed the specified value  $\varepsilon^*$ , will be considered as the reliability of the obtained estimate  $p[|r(x) - \overline{r}| \le \varepsilon^*] = \alpha$ .

Using the introduced notation, the formulas for estimating the error and the required number of experiments will be as follows:

$$
\varepsilon = t_{\alpha} \sigma / \sqrt{N}, \quad N^* = t_{\alpha}^2 \sigma^2 / \varepsilon^2, \tag{20}
$$

where  $t_{\alpha}$  – table parameter (quantile of the normal probability distribution for a given confidence level  $\alpha$ );  $\sigma$  – standard deviation of the time destribution and execution of the work package  $\bar{\tau}$  estimation.

In order to reduce the variance of the obtained estimates, it is proposed to discard the initial statistics while maintaining the state of the channels and the occupancy of the respective queues.

It is proposed to determine the conditions for automatic stopping of the model experiment using a two-stage procedure. At the first stage, experiments are performed to estimate the required number of experiments  $N^*$  (20). If  $N \ge N^*$ , then the required accuracy of the results has been achieved (20), otherwise,  $N \geq N^*$ experiments need to be performed at the second stage.

The software implementation of the proposed algorithm was carried out using a statistical simulation modelling software package. The results of the experiments made it possible to determine the characteristics of technological facilities as queuing systems with different numbers of channels and different laws of distribution of the flow of work packages and the time of execution of various works.

### **Conclusions**

Based on the results of the review and analysis of the current state of the problem of management and reengineering of technological systems, it is established that the models of the classical problem of work package distribution in many cases do not meet the requirements of practice: the goal may be to find the maximum of the objective function (profit, quality of work, etc.); there may be several objective functions; the number of works in packages may not be equal to the number of executors; there may be prohibitions on the assignment of some works to certain executors; material (financial), time costs, and other parameters of the process of functioning of the facility.

Taking into account the dynamics of the incoming flow of work packages, incomplete certainty of the parameters of the facility's functioning process, and the qualifications of executors, it is necessary to jointly solve the problems of determining the workload of executors and the distribution of work. In view of this, a solution to the scientific and applied problem of increasing the efficiency of technological systems is proposed by developing an analytical and simulation model of the cyclic distribution of work packages by a set of indicators, taking into account the workload of executors and incomplete certainty of process parameters.

A model of the problem of distribution of work packages by indicators of material (financial) costs, system performance and quality of work performance by elements with different functional and cost characteristics is proposed. Taking into account the probabilistic nature of the input flow of work packages and the time of their execution, it is proposed to determine the system performance using simulation statistical modelling of the technological system as a three-phase multi-channel queuing system. The tasks of tactical planning of computer experiments have been solved, which allow obtaining results of the required accuracy and reliability.

The developed models help to determine the characteristics of technological systems with an unequal number of elements and different laws of distribution of packet flow and time of various works. Practical application of the proposed models will allow to obtain more efficient solutions to the problems of packet distribution, taking into account the occupancy of channels and the cost of their reconfiguration after the previous work.

Areas for further research include: developing a technology for implementing the proposed tools for modelling the processes of distribution and execution of work packages in control systems, design or reengineering of technological systems; creating mathematical models and decision support methods for optimising technological systems in conditions of incomplete certainty of costs and preferences of the decision maker.

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