DOI: https://doi.org/10.30837/MMP.2020.316

NUMERICAL CRITICAL IDENTIFICATION PROCEDURE

Timofeyev V., Jakushik I., Khrustalev K., Khrustalova S.

The article discusses the solution to the problem of monitoring and control of objects under arbitrary, but limited external disturbances.

A computational procedure based on the target inequality method is proposed, the advantage of which is the simplicity of computational implementation. The research results can be used to solve a wide class of monitoring and control problems.

Introduction

In monitoring and control tasks, a situation often arises when a closed monitoring and control system, which is under the influence of an external disturbing signal (external, reference signal, interference, signals from other objects, variations in environmental parameters, etc.), must maintain the characteristics control object (object output signal, control error, etc.) within some a priori set boundaries so that

$$|v(t,w)| \le E \quad \forall t \in R, \tag{1}$$

where t - is continuous or discrete time. In the event that violation of inequality (1) is in principle unacceptable, for example, leads to catastrophic consequences, the control law that ensures rigid maintenance (1) is called critical, and the control system that implements it is called critical [1].

In everyday practice, critical management tasks are encountered quite often, and among the most characteristic are the following:

- in the tasks of air traffic control, the aircraft must constantly be inside a rather narrow air corridor, leaving the boundaries of which, in principle, is not permissible];

- a catalytic converter, the presence of which in a car is required by the legislation of most civilized states, works effectively only in situations where the characteristics of the air-fuel mixture are maintained within tight boundaries;

- in telecommunication systems, the tracking accuracy by the communication satellite system is set in the form of a narrow error range;

- in biomedical systems, the control parameters of the controlled organism must be within the boundaries that guarantee stable vital activity.

In the general case, the problem of maintaining the output signals of the control object within the given boundaries arose quite a long time ago and a number of approaches were developed to solve it. So in [2], a statistical approach was proposed that maximizes the probability that the outputs of the object will not go beyond certain boundaries for arbitrary random inputs. There is a known method based on the set-theoretic approach [3], using the concept of a "target tube", inside which the phase variables of an object under the influence of unknown but restrictive disturbances must remain. An efficient computational algorithm implementing this approach was proposed in [4], and the solution to the problem was extended to nonlinear multidimensional objects.

This article discusses a group of numerical algorithms that implement various procedures and techniques for critical control. This is, first of all, the procedure of moving boundaries. The main requirements that determined the choice of this particular approach are accuracy, ease of implementation and the possibility of working in real time.

In the general case, the goal of any feedback control system is to ensure the required behavior of the object by appropriate processing of input and output signals, calculation of control actions and their delivery to the executive bodies. The main problem in this case is the projection of the regulator itself, from a theoretical point of view, it is a formal algorithm, the result of which is the numerical value of the control signal.

In design usually considers many criteria and subgoals, many of which are competing or even contradictory. Therefore, when designing, it is extremely important to be able to take into account the trade-off between different criteria. It is important to note that although many different criteria have been proposed within the framework of control theory, there is no universal criterion that takes into account all possible requirements for the quality of processes occurring in the system. Therefore, the developer of the monitoring and control system must choose a criterion or criteria that take into account his often-subjective ideas about how this system should behave. Moreover, for an arbitrary criterion chosen at random, there is always a control algorithm that provides an extremum for this criterion. In practice, however, usually used criteria related to the accuracy of regulation or tracking, efficiency in terms of speed, performance, noise immunity, costs, stability, etc. These criteria usually represent some functions of the input and output signals or states of the control object, while the signals are usually assumed to be stochastic processes with some a priori given probabilistic structure. The most commonly used hypothesis is the Gaussian distribution of useful signals and interference. It is on this hypothesis that the popular LCG problem], the H^2-optimization problem and the classical stochastic control theory [5,6] are based. In practical problems, Gaussian processes are not so common, and their use is mainly associated with mathematical convenience.

An optimization problem H^∞ is associated with less restrictions on the statistical nature of signals, which requires only that the signals of the object be square integrable and have limited energy.

Greater flexibility in the design of control systems can be achieved by assuming that the signals belong to a certain functional space. This space can be determined by setting boundaries for the amplitudes, rate of change, energy, and other characteristics of the signals. Such a description of signals is much simpler than a statistical one, has a clear physical meaning and, in general, facilitates the process of designing a monitoring and control system.

Main part

The inequality method can be implemented using various numerical algorithms, the most effective of which, in our opinion, is the procedure of moving boundaries [7].

So, let the system of inequalities be given

$$J_i(p) \le \varepsilon_i \quad \forall\ i = 1, 2, \dots, n, \tag{2}$$

where ε_i are real numbers, $p=(p_1,p_2,...,p_n)^T$ is a vector whose elements are parameters of the synthesized controller; J_i - are real functions of p, which are either control criteria or constraints imposed on phase variables.

Each inequality $J_i(p) \le \varepsilon_i$ of system (2) defines a set of points S_i in N-dimensional space R^N such that

$$S_i = \{p : J_i(p) \le \varepsilon_i\}. \tag{3}$$

The boundary of this set is determined by the equation

$$J_i(p)=\varepsilon_i. \tag{4}$$

A point $p \in R^N$ is a solution to system (2) only if it belongs to each of S_i, i.e. crossing

$$S = \bigcap_{i=1}^{n} S_i. \tag{5}$$

The moving boundaries procedure is a recurrent algorithm that provides iterative motion from an arbitrary starting point p^0 to some feasible point p_s belonging to the set (5).

Let us denote p^k the state of the vector of parameters at the k-th cycle of calculations, and S_i^k - the set formed by the inequality

$$J_i(p) \le J_i(p^k), \tag{6}$$

and bounded by the equation

$$J_i(p) = J_i(p^k). \tag{7}$$

A test step is carried out from a point p^k to a certain point $\tilde{p}^k$, while, if for each $i = 1, 2, \dots$

$$J_i(p)=J_i(\tilde{p}^k) \tag{8}$$

closer to (4) border S_i^k, then the point $\tilde{p}^k$ is taken as a new point p^{k+1}. After a series of successful steps the boundary S_i^k coincides with S_i^k for each $i=1,2,...,n$, which gives a solution to the problem. Formally, this procedure can be written as follows:

$$S^k = \bigcap_{i=1}^{n} S_i^k, \tag{9}$$

$$S_i^k = \{p : J_i(p) \le \varepsilon_i^k\}, \qquad i = 1, 2, ..., n, \tag{10}$$

$$\varepsilon_i^k = \begin{cases} \varepsilon_i, \text{ if } J_i(p^k) \le \varepsilon_i, & i = 1, 2, ..., n, \\ J_i(p^k) & \textit{otherwise}, \end{cases} \tag{11}$$

the trial point is recognized successfully

$$p^{k+1} = \tilde{p}^k \tag{12}$$

only in case

$$J_i(\tilde{p}^k) \le \varepsilon_i^k, \qquad i = 1, 2, ..., n. \tag{13}$$

if any of inequalities (13) fails, a new test point $\tilde{p}^k$ is described and the procedure is repeated again. With each new successful point p^{k+1}, the border S^{k+1} is pulled closer and closer to the border of the admissible set S. The iterative process continues until the boundaries S^k converge to S, i.e. until the condition is met

$$\varepsilon_i^k = \varepsilon_i, \qquad i = 1, 2, ..., n. \tag{14}$$

In principle, any of the nonlinear programming algorithms can be used to find sample points; although from a computational point of view, the scheme proposed by Rosenbrock seems to be preferable.

Let us introduce a system of orthonormal vectors $V^r = 1,2,...,N$ and the corresponding step parameters e_j. The sample point is calculated according to the relation

$$\tilde{p}^k = p^k + e_j V_j^r, \tag{15}$$

in this case, if $\tilde{p}^k$ successful, it is e_j replaced by $3e_j$. Otherwise ($\tilde{p}^k$ unsuccessful) e_j is replaced with $-0.5\,e_j$. If the test is successful, j it is replaced by $j+1$ and (15) is repeated. One iteration of the procedure consists of N samples, after the last of which ($j=N$) is assumed $j=1$ and $j=1$, i.e. proceeds to the next step. If successful steps alternate with unsuccessful ones, the vector V^r will be replaced V^{r+1} $(j=1,2,...,N)$, which is calculated as follows.

We introduce a parameter d_j equal to the sum of successful values e_j at the r -th stage and calculate

$$\begin{cases} a_1 = d_1 V_1^r + d_2 V_2^r + \ldots + d_N V_N^r, \\ a_2 = d_2 V_2^r + \ldots + d_N V_N^r, \\ \cdot \\ \cdot \\ \cdot \\ a_N = d_N V_N^r, \end{cases} \tag{16}$$

after which we orthogonalize the system of vectors a_j using the Gram-Schmidt procedure:

$$\begin{cases} b_1 = a_1, \quad V_1^{p+1} = \dfrac{b_1}{\| b_1 \|}, \\ b_2 = a_2 - a_2^T V_1^{r+1} V_1^{r+1}, \quad V_2^{r+1} = \dfrac{b_2}{\| b_2 \|}, \\ \cdot \\ \cdot \\ \cdot \\ b_N = a_N - \sum_{k=1}^{N-1} a_N^T V_k^{r+1} V_k^{r+1}, \quad V_N^{r+1} = \dfrac{b_N}{\| b_N \|}. \end{cases} \tag{17}$$

Thus, at the initial stages at $r=0$, e_j and V_j^T are chosen rather arbitrarily. However, with an increase r, the rate of convergence S^k to S increases, since the vectors V_j^T are oriented in the directions of maximum change in the tuned parameters p.

It is convenient to write this procedure as the following sequence of steps.

1. Set limits $\varepsilon_i \quad (i=1,2,...,n)$ and maximum number of iterations N_m.

2. Set $p=p^0$. Calculate $J_i(p) \quad (i=1,2,...,N)$. If $J_i(p) \le \varepsilon_i \quad \forall i$, then the starting point satisfies all constraints, i.e. is valid. The problem has been resolved.

3. Set $e_j^0=0,1|p_j|$ if $|p_j| \ge 0,1 \, (j=1,2,..,N)$, set $e_j^0=0,01$ if $|p_j| \ge 0,1 \, (j=1,2,..,N)$.

Set

$V_1=(100...000)^T$,

$V_2=(010...000)^T$,

$V_{N-1}=(000...010)^T$,

$V_N=(000...001)^T$.

Put $\varepsilon_i^{'}=\varepsilon_i$ if $J_i(p)\le\varepsilon_i$ $(i=1,2,...,n)$.

Put $L=0,\ r=0$.

4. Put $e_j=e_j^0$ and $d_j=0$.

5. Start a new iteration. Put $L=L+1, j=1$.

6. Generate test point $\tilde{p}=p+e_jV_j$.

Calculate $J_i(p)$ $(i=1,2,...,n)$.

Check inequality $J_i(p)\le\varepsilon_i^{'}$ $(i=1,2,...,n)$.

If successful, go to 7.

If unsuccessful, go to 8.

7. Put $p=\tilde{p},\ d_j=d_j+e_j,\quad e_j=3e_j$.

Put $\varepsilon_i^{'}=J_i(p)$ if $J_i(p)>\varepsilon_i$ $(i=1,2,...,n)$.

Put $\varepsilon_i^{'}=\varepsilon_i$ if $J_i(p)\le\varepsilon_i$ $(i=1,2,...,n)$.

Check equality $\varepsilon_i^{'}=\varepsilon_i$ $(i=1,2,...n)$.

If the condition is met, the problem is solved. Otherwise, go to 9.

8. Drop $\tilde{p}$ and put $e_i=-0.5e_i$. Count the successes and failures for each V_i Reinitialize vectors V_i $(i=1,2,...,N)$ according to expressions (16) and (17), set $r=r+1$ and go to step 4.

9. If $j=N$, go to item 5.

If $j<N$, put $j=j+1$ and go to 6.

Conclusions

The article discusses the solution to the problem of monitoring and controlling objects under arbitrary, but limited external disturbances.

A computational procedure based on the target inequality method is proposed. The research results can be used to solve a wide class of monitoring and control problems.

REFERENCES

1. Timofeyev, V. (2004), "On some problems of synthesis of critical control systems", *Radio electronics. Informatics. Control*, No. 1 (11), P. 160–162.
2. Simankov, V. S., Khalafyan, A. A. (2009), *System analysis and modern information technologies in medical decision support systems*, Moscow, BinomPress, 362 p.
3. Shelestov, A., Kussul, N. (2008), "Using The Fuzzy-Ellipsoid Method For Robust Estimation Of The State Of a Grid System Node", *Cybernetics and System Analysis*, Vol. 44, No. 6, P. 847–854.
4. Usoro, P. B., Schweppe, F. C., Wormley, D. N., Gould, J. A. (1982), "Ellipsoidal set-theoretic control synthesis", *ASME J. Dyn. Syst. Meas. & Control*, No. 104, P. 331–336.
5. Bacastov, T., Kussul, N. (2009), "Redefining geospatial intelligence", *American Intelligenct Journal*, P. 38–40.
6. Boguslaev, A. V., Oleinik, Al. A., Oleinik, An. A., Pavlenko, D. V., Subbotin, S. A. (2009), *Progressive technologies for modeling, optimization and intelligent automation of the stages of the life cycle of aircraft engines* : Monograph, Zaporozhye, Motor Sich OJSC, 468 p.
7. Timofeyev, V. (2003), "On one approach to the synthesis of a critical regulator dynamic system", *Automobile Transport*, Issue 13, P. 269–271.