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Quantum Entropy in Machine Learning Tasks WS

Victor Syneglazov[1†] and Petro Chynnyk[2*†]

[1] *National Aviation University, Lubomyr Huzar avenue 1, 03058 Kyiv, Ukraine*
[2] *NTUU "Igor Sikorsky Kyiv Polytechnic Institute", 37 Beresteysky prospect, 03056 Kyiv, Ukraine*

Abstract
This paper investigates quantum entropy as a function of error for model training. A comparative analysis of the quality of models trained using quantum entropy with models trained based on standard error functions: cross-entropy and MSE was performed. The results demonstrate an advantage in the quality and training time of the models that used quantum entropy for training, compared to those models that used classical functions.

Keywords
quantum machine learning, quantum entropy, loss function, cross-entropy, mse

1. Introduction

In recent years, quantum computing has become a promising frontier that could change the way we process information. Improve artificial intelligence models, improve information protection, speed up modeling of complex natural processes.

If we consider the use of artificial intelligence in quantum computing, one of the more difficult problems is learning. Today, classical error functions such as cross-entropy and MSE are used to train quantum models of artificial intelligence. We suggest using another function, namely quantum entropy.

Derived from quantum information theory, quantum entropy offers a probabilistic framework that captures uncertainty in ways not accessible through classical measurements. Incorporating such quantum principles into the design of error functions for machine learning models opens up new ways to optimize learning processes and increase efficiency.

The main goal of this work is to perform a comparative analysis of models trained by quantum entropy with models trained by classical functions such as cross-entropy and MSE. In particular, we investigate whether the introduction of quantum entropy can lead to improvements in convergence speed, model accuracy, and overall quality. By investigating the behavior of quantum entropy as a function of error, we aim to find out whether it offers measurable advantages over traditional optimization strategies.

2. Quantum entropy

Quantum entropy is an extension of the concepts of entropy for quantum systems. Its development began within the framework of quantum mechanics and quantum information in order to describe uncertainty and mixed states in quantum systems.

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* Corresponding author.
† These authors contributed equally.
svm@nau.edu.ua (V. Syneglazov); chynnyk@vivaldi.net (P. Chynnyk)
0000-0002-3297-9060 (V. Syneglazov); 0009-0002-6787-5429 (P. Chynnyk)

The foundations were laid in 1932 by John von Neumann, who first introduced the von Neumann entropy. This entropy generalizes the classical Shannon entropy to the case of quantum states (1).

$$S(\rho) = -Tr\left(\rho log(\rho)\right) \tag{1}$$

where ρ – density matrix.

In order to translate the quantum entropy into an error function, it is necessary to introduce a reference density matrix that will represent the class of the object. This reference density matrix must represent a pure quantum state.

For binary classification, we propose these reference density matrices (2,3)

$$\rho = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \tag{2}$$

$$\rho = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \tag{3}$$

The Scaling and Squaring Algorithms algorithm was used to calculate the logarithm from the matrix.

For learning the quantum machine learning model we suggest the next loss function

$$L(\theta) = \frac{1}{N}\sum_{i=1}^{N} -Tr\left(\rho_i log\left(\rho_i(\theta)\right)\right) \tag{4}$$

where N – number of objects, θ – parameters of model.

3. Result of experiments

For training, we used the iris dataset, from which we selected two classes. We selected 70 objects for the training sample, 30 for the test sample.

The quantum model was as follows (Fig. 1)

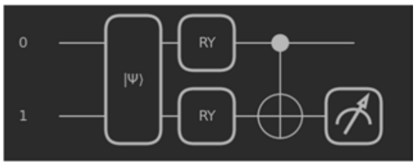

Figure 1: Quantum circuit of quantum machine learning model

The training was carried out using Adam's algorithm.

Result of learning show in Table 1

Table 1
Result of learning

Optimize	Type Loss	Train	Test	Accuarcy Train	Accuaracy Test	F1 Train	F1 Test
MSE	MSE	0,4748	0,50203	1	1	1	1
	Cross Entropy	0,41924	0,43402				
	Quantum Entropy	0,45703	0,4955974				

Optimize	Type Loss	Train	Test	Accuracy Train	Accuracy Test	F1 Train	F1 Test
Cross Entropy	MSE	0,54974	0,56639	1	1	1	1
	Cross Entropy	0,46166	0,470315				
	Quantum Entropy	0,60931	0,67563				
Quantum Entropy	MSE	0,45424	0,45344	1	1	1	1
	Cross Entropy	0,40953	0,40893				
	Quantum Entropy	0,42522	0,42784				

Below are graphs of model training and graphs of other error functions based on model training

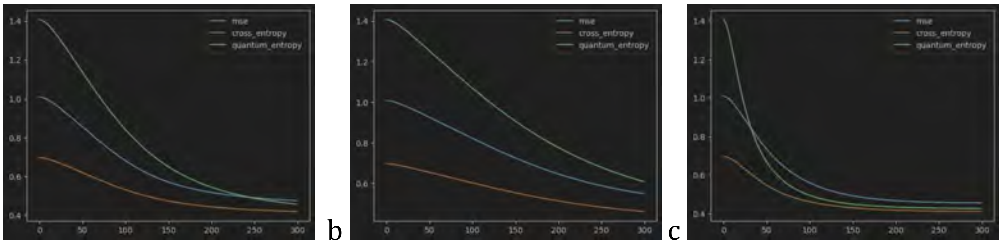

Figure 2: Graphic of learning of model. a - MSE, b – cross-entropy, c – quantum entropy

4. Conclusion

As we can see from the experimental results, all models show excellent prediction results regardless of the type of loss function. As we can see from graph 2, the quantum entropy loss function shows better results.

The error function used for training, quantum entropy, trains the model faster than MSE or cross-entropy. Already at 100 iterations, the error value is smaller than that of MSE. And the value of MSE itself is smaller than when using MSE for training.

Learning with quantum entropy is better because it better takes into account the very nature of quantum computing and exploits the implicit, hidden patterns between qubits.

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